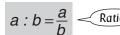
14. [Rates / Ratios]

Skill 14.1 Simplifying ratios by comparing two numbers.

EITHER

- Find the largest number that divides evenly into each number of the ratio (Greatest Common Factor).
- Divide each number by the GCF.

Hint: ':' means fraction and is read as 'to'.



OR

Divide each number of the ratio by any factor until the ratio is reduced to simplest form.

Q.	Simplify the ratio	32:56	A. $32:56$ 0R $32:560R$ 0 0 0 0 0 0 0 0 0 0	A. $32:56$ $\div 2(16 28)$ = 32:56 Simplify: $\div 2$
			= 4 :7	= 16:28
				$= 8:14$ $\div 2 \begin{pmatrix} 8:14\\ 4&7\\ =&8:14 \end{pmatrix}$ $= 4 \cdot 7$ Simplify: $\div 2$

a) Simplify the ratio 4:6

$$= \overset{2}{\cancel{A}} : \overset{3}{\cancel{B}} = 2:3$$

b) Simplify the ratio 6:12

$$= 6:12$$
 Simplify: \div 6
$$=$$
 \vdots

Simplify the ratio 30:50

d) Simplify the ratio 10:15

e) Simplify the ratio 45:15

f) Simplify the ratio 18:24

g) Simplify the ratio 100:70

h) Simplify the ratio 32:8

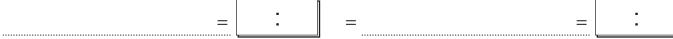
Simplify the ratio 24:96

Simplify the ratio 30:54



k) Simplify the ratio 27:36

Simplify the ratio 24:16



m) Simplify the ratio 150:45

- n) Simplify the ratio 90:240

Skill 14.2 Simplifying ratios by comparing two quantities.



Write the quantities of the ratio with the same unit of measurement.

EITHER

- Find the largest number that divides evenly into each quantity of the ratio (Greatest Common Factor).
- Divide each quantity by the GCF.

Hints: The order of the quantities in a ratio matters.

"" means fraction and is read as "to".

Examples: The ratio of legs to ears in a cat is 4:2=2:1

The ratio of ears to legs in a cat is 2:4=1:2

$$a:b=\frac{a}{b}$$
 Ratio

Divide each quantity of the ratio

by any factor until the ratio is

reduced to simplest form.

Q. Simplify the ratio 10 oz : 2 lb

10 oz : 2 lb

$$\begin{array}{c}
\stackrel{\div}{=} 2 \begin{pmatrix} 5 \\ -2 \end{pmatrix} \begin{pmatrix} 16 \\ 02 \end{pmatrix} \begin{pmatrix} 16 \\ 32 \end{pmatrix} \begin{pmatrix} 16 \\ 02 \end{pmatrix} \begin{pmatrix} 16 \\ 2 \end{pmatrix} \begin{pmatrix}$$

OR

a) Simplify the ratio 48 kg: 80 kg

$$= 48 : 80$$
 Simplify: ÷ 16 =

b) Simplify the ratio 50 ft: 125 ft

c) Simplify the ratio 120 yd: 36 yd

d) Simplify the ratio 150 in.: 175 in.

e) Simplify the ratio \$3.00 : 40 cents

\$3.00 = 3.00 \times 100\epsilon = 3.00 \times 100\epsilon = 3.00\epsilon \$2 zeros, 2 places right

$$3.00 = 3.00 \times 100 = 3.00 =$$

$$=3\theta 0 \phi : 4\theta \phi \qquad \qquad = \boxed{15:2}$$

f) Simplify the ratio 40 s : 2 min

g) Simplify the ratio 6 yd: 6 ft

h) Simplify the ratio \$4.00 : 25 cents

Simplify the ratio 6 days: 4 weeks j) i)

Simplify the ratio 3 ft: 8 in.

Skill 14.3 Solving questions involving distance, time and speed (1).



$$\text{rate (speed)} = \frac{\text{distance traveled }(d)}{\text{time taken }(t)} \quad \text{OR} \quad r = \frac{d}{t}$$

$$\text{distance traveled }(d) = \text{rate }(r) \cdot \text{time taken }(t) \quad \text{OR} \quad d = rt$$

$$\text{time taken }(t) = \frac{\text{distance traveled }(d)}{\text{rate }(r)} \quad \text{OR} \quad t = \frac{d}{r}$$

- Write the formula for the rate (speed), or distance or time taken to travel.
- Convert the given units into the required units if necessary. (see Math Facts, page 377)
 Hints: If the rate (speed) must be calculated in mph, convert the units for distance to miles and the units for time to hours.

Changing from smaller units into larger units, always divide by the conversion factor. Changing from larger units into smaller units, always multiply by the conversion factor.

- Substitute the known values into the formula.
- Simplify and evaluate.
- **Q.** A jet travels at an average speed of 550 mph. How long would it take to travel 2475 miles?

A.
$$t = \frac{distance traveled}{rate} = \frac{d}{r}$$

$$= \frac{2475 \text{ mi}}{550 \text{ mph}}$$
Substitute into the formula
$$= \frac{2475}{550} h$$
Simplify: ÷ 25
$$= \frac{999}{222} h$$
Simplify: ÷ 11
$$= 4.5 h$$

a) How far will John walk in 45 minutes if he walks at 10 km/h?

$$t = 45 \text{ min} = 0.75 \text{ h} \text{ (three quarters of an hour)}$$

$$d = \text{rt}$$

$$d = 10 \text{ km/h} \cdot 0.75 \text{ h} = 7.5 \text{ km}$$

A cyclist rides at an average speed of 18 km/h. How long would it take to travel 45 km?

travel 45 km?
$$t = \frac{545 \text{ km}}{2 \text{ J8 km/h}} = \frac{\text{Simplify: } \div 9}{\text{Simplify: } \div 9}$$

$$= \frac{\text{h}}{2 \text{ simplify: } \div 9}$$

c) The X-15 rocket plane is the fastest aircraft with a maximum speed of 4520 mph, reached in 1967. At this rate how far could it travel in 5 hours?

mi

d) An airplane flew from Denver to New York, a distance of 1600 miles. If the plane traveled at an average speed of 500 mph, how long did the trip take?

Skill 14.3 Solving questions involving distance, time and speed (2).



e) An airplane flew from Memphis to Pittsburgh, a distance of 660 miles. If the plane traveled 1.1 hours, how fast did it travel?

$$r = \frac{d}{t} = \frac{{}^{60}660 \, \text{mi}}{{}^{0.1} \, \text{M} \, h} = 60 \div 0.1 \, \text{mph}$$

$$=600 \div 1 \, mph$$
 $=$ mph

An airplane flew from Houston to Salt Lake City, a distance of 1200 miles. If the plane traveled 2.5 hours, how fast did it travel?

$$r =$$

g) An emu can run 9 km in 12 minutes. What is its average speed in kilometers per hour?

9 miles at 36 mph. How long would it take to swim this distance?

$$t =$$
 $=$
 min

i) A train travels at an average speed of 48 mph. What distance would it travel in one hour and 15 minutes?



j) A satellite orbits the earth at an average speed of 8 km/s. What distance does it travel in 20 minutes?

$$d =$$
 $=$
 km

k) How far will a salmon swim in 12 minutes if it swims at 45 km/h?

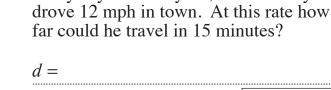
$$a =$$
 $=$
 km

A hot air balloon travels at a speed of 21 km/h. At this rate how far will it travel in 40 minutes?

$$d =$$
 $=$
 $=$
 km

n) In 1904 the first speeding ticket went to Harry Myers of Dayton, Ohio. Harry

m) Earth moves around the sun at an average speed of 65,000 mph. What distance does it move in a quarter of an hour?



$$d =$$
 $=$
 mi

Simplifying ratios by comparing three numbers. Skill 14.4



EITHER

- Find the largest number that divides evenly into each number of the ratio (Greatest Common Factor).
- Divide each number by the GCF.

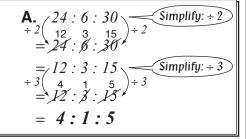
OR

Divide each number of the ratio by any factor until the ratio is reduced to simplest form.

Q. Simplify the ratio 24:6:30

A.
$$(24:6:30)$$

 $\div 6(4 1 5)$
 $= 24:6:30$
 $= 4:1:5$



a) Simplify the ratio 72:16:40

$$= \frac{\cancel{9}}{\cancel{72}} : \cancel{\cancel{16}} : \cancel{\cancel{40}} = \boxed{9:2:5}$$

c) Simplify the ratio 3:9:27

e) Simplify the ratio 33: 18: 15 **f)** Simplify the ratio 18: 36: 27

g) Simplify the ratio 48:18:12

i) Simplify the ratio 36:12:60 j) Simplify the ratio 30:45:90

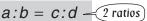
k) Simplify the ratio 42:14:21

Skill 14.5 Deciding if two ratios form a proportion.



Write the two ratios as equal fractions side by side.





- Cross multiply the numerators and the denominators of the fractions.
- If the two products are equal, then the two ratios are in proportion.
- $a \cdot d = b \cdot c$ ad = bc

- **Q.** Which ratio forms a proportion with $\frac{5}{7}$?

$$\frac{5}{7} \times \frac{10}{35} \Rightarrow 5 \cdot 35 = 7 \cdot 10$$

A) $\frac{10}{25}$ B) $\frac{15}{14}$ C) $\frac{20}{28}$

Cross multiply

$$\frac{5}{7} \times \frac{15}{14} \Rightarrow 5 \cdot 14 = 7 \cdot 15
\Rightarrow 70 = 105 \Rightarrow false$$

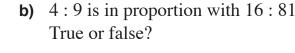
$$\frac{5}{7} \times \frac{20}{28} \Rightarrow 5 \cdot 28 = 7 \cdot 20$$
$$\Rightarrow 140 = 140 \Rightarrow true$$

The answer is C.

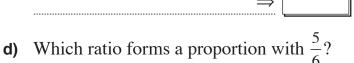
a) 5: 12 is in proportion with 25: 60 True or false?

$$\frac{5}{12} \times \frac{25}{60} \qquad \Rightarrow 5 \cdot 60 = 12 \cdot 25$$

$$300 = 300$$



300 = 300



- c) Which ratio forms a proportion with $\frac{3}{5}$?
 - A) $\frac{9}{25}$ B) $\frac{9}{15}$ C) $\frac{18}{25}$

A)
$$\frac{10}{30}$$
 B) $\frac{25}{36}$ C) $\frac{35}{42}$

- $A \xrightarrow{3} \times \frac{9}{25} \qquad \Rightarrow 3 \cdot 25 = 5 \cdot 9 \Rightarrow 75 = 45 \ (F)$

В

 $B \frac{3}{5} = \frac{9}{15} \implies$

C

- C

- e) Which ratio forms a proportion with $\frac{2}{7}$?
- A) $\frac{8}{28}$ B) $\frac{10}{70}$ C) $\frac{4}{40}$

- f) Which ratio forms a proportion with $\frac{1}{9}$?
- A) $\frac{21}{36}$ B) $\frac{49}{81}$ C) $\frac{35}{45}$

- \boldsymbol{A}

- \boldsymbol{A}

- B

- B

Skill 14.6 Finding the missing term in a proportion (1).

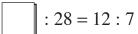


Write the proportion as two equal fractions.

- (A proportion) a:b=c:d-2 ratios
- Cross multiply the numerators and the denominators of the fractions.

Equate the products.

- $a \cdot d = b \cdot c$ (Cross product) ad = bc
- Solve the equation to find the missing number (x).
- **Q.** Complete the missing term in the proportion:



- **A.** $\frac{x}{28} = \frac{12}{7}$ $\frac{x}{28} + \frac{12}{7}$ Cross multiply $x \cdot 7 = 28 \cdot 12$ $7x = 28 \cdot 12$ $x = 4 \cdot 12$ x = 48
- a) Complete the missing term in the proportion:

$$3: \boxed{4} = 12:16$$

$$\frac{3}{x} \times \frac{12}{16} \Rightarrow 3 \cdot 16 = x \cdot 12$$

$$\frac{1}{12} \times \frac{1}{12} = \frac{13 \cdot 16}{12} \xrightarrow{\text{Simplify: } \div 3. \div 4} \Rightarrow x = 4$$

b) Complete the missing term in the proportion:

c) Complete the missing term in the proportion:

: 90

 $\Rightarrow x =$

 $\Rightarrow x =$

$$\frac{5}{9} = \frac{35}{2}$$

3:10=

Complete the missing term in the f) proportion:

$$\frac{3}{7} = \frac{18}{1}$$

$$\Rightarrow x =$$

$$\Rightarrow x =$$

Skill 14.6 Finding the missing term in a proportion (2).



g) Complete the missing term in the proportion:

$$\frac{1}{7} = \frac{1}{56}$$

 \Rightarrow

 $\Rightarrow x =$

h) Complete the missing term in the proportion:

$$\frac{7}{20} = \frac{}{140}$$

 \Rightarrow

 $\Rightarrow x =$

i) Complete the missing term in the proportion:

$$\frac{24}{40} = \frac{3}{100}$$

 \Rightarrow

 $\Rightarrow x =$

j) Complete the missing term in the proportion:

$$\frac{20}{15} = \frac{4}{15}$$

 \Rightarrow

 $\Rightarrow x =$

k) Complete the missing term in the proportion:

$$\frac{10}{45} = \frac{10}{9}$$

 \Rightarrow

 $\Rightarrow x =$

I) Complete the missing term in the proportion:

$$\frac{64}{80} = \frac{10}{10}$$

=

 $\Rightarrow x =$

m) Complete the missing term in the proportion:

$$\frac{24}{15} = \frac{24}{5}$$

 \Rightarrow

 $\Rightarrow x =$

n) Complete the missing term in the proportion:

$$\frac{11}{5} = \boxed{\frac{1}{15}}$$

=

 $\Rightarrow x =$

Skill 14.7 Solving word problems involving proportions (1).

To decide which deal is cheaper:

EITHER

- Find the unit price for each case, by dividing the cost price by the number of units.
- Compare the results.

To solve word problems involving proportions: EITHER

- Find the unit rate, by dividing the amount by the given number of units.
- Multiply the unit rate by the required number of units.

OR

 Use any other method to make the cost price the same or the number the units the same for both deals.

OR

- Write a proportion using words.
- Replace the words with numbers:

First the given ratio.

Then the ratio of the given quantity to the unknown quantity.

• Find the missing term of the proportion. (see skill 14.6, page 109)

- **Q.** Which is cheaper per soda can?
 - A) \$2.50 for a 6-pack
 - B) \$6 for a 12-pack
- **A.** Deal A)

 \$2.50 \$0

$$\frac{\$2.50}{6 \ cans} = \frac{\$0.42}{1 \ can}$$

 \Rightarrow unit price = \$0.42

Deal B)
$$\frac{\$6.00}{12 \text{ cans}} = \frac{\$0.50}{1 \text{ can}}$$

 \Rightarrow unit price = \$0.50

Deal A) is cheaper.

OR A. Make the same number of cans:

Deal A) double the quantity \Rightarrow double the cost

 $$2.50 \text{ for } 6 \text{ cans} \Rightarrow$

\$5.00 for 12 cans

Deal B)

 $$6.00 for 12 cans \Rightarrow$

Deal A) is cheaper.

a) There are 110 calories in 50 g of white bread. How many calories in 80 g?

calories : bread = calories : bread110 : 50 = x : 80

$$\frac{110}{50} \times \frac{x}{80} \implies 110 \cdot 80 = 50 \cdot x$$

$$x = \frac{110 \cdot \cancel{80}}{\cancel{50}}$$

$$x =$$

176 cal

b) If 5 yd of curtain material costs \$65, how much would 9 yd cost?

yd: dollars = yd: dollars

 \Rightarrow

x =

x =

c) If 80 concert tickets cost \$360, how much would 30 tickets cost?

80 tickets for \$360

d) If 9 pens cost \$22.50, how much would 6 pens cost?

9 pens for \$22.50

- \Rightarrow 10 tickets for \$360 ÷ 8 = \$45
- \Rightarrow 30 tickets for \$45 \times 3 =
- \$

=

Skill 14.7 Solving word problems involving proportions (2). MMBlue 11 22 33 4, MMGreen 11 22 3 5 44						
e)	There are 300 calories in 4 oz of pork chops. How many calories in 10 oz?	f)	If 10 lb of minced beef costs \$35, how much would 6 lb cost?			
	4 oz have 300 cal		10 lb for			
\Rightarrow	2 oz	=	2 lb			
\Rightarrow	10 oz = ca			= \$		
g)	If a car travels 180 miles on 8 gallons, how far does it travel on 12 gallons at the same rate?		There are 240 calories in 5 c steak. How many calories is			
\rightarrow		⇒				
<i>→</i>	= m	 :	=	cal		
	Which is cheaper per card? A) \$4 for 12 cards B) \$6 for 15 cards $\times 3$ Make the same cost $\times 3$ \$4 for 12 cards \Rightarrow \$12 for 36 cards	j) A		e the same cost		
<i>B</i>)	$\$6 \text{ for } 15 \text{ cards} \stackrel{\times 2}{\Rightarrow} \$12 \text{ for } 30 \text{ cards} \Rightarrow \triangle$	B_{i})	⇒		
k)	Which is cheaper per ounce? A) \$20 for 10 oz B) \$40 for 22 oz	I)	Which is cheaper per ounce A) \$16 for 15 oz B) \$27 for 25 oz	?		
A)		A,)			
<i>B</i>)	\Rightarrow	B_{c})	⇒ <u></u>		
m)	Which is cheaper per apple? A) \$4.80 for 4 apples B) \$6.50 for 6 apples	n)	Which is cheaper per yard? A) \$25 for 12 yards B) \$40 for 17 yards			
B)						
D)	→ <u> </u>	<i>D</i>		<u> </u>		

Skill 14.8 Finding the ratio of two quantities (1).



- Write the ratio in words.
- Replace the words with numbers.
- Simplify the ratio:

EITHER

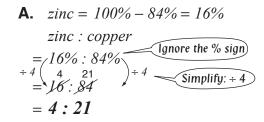
 Find the largest number that divides evenly into each quantity of the ratio (Greatest Common Factor) and divide each quantity by the GCF.

Hint: The order of the quantities in a ratio matters.

OR

 Divide each quantity of the ratio by any factor until the ratio is reduced to simplest form.

Q. The common metal for medals is 84% copper, and the rest is zinc. Find the ratio of zinc to copper.



length: diagonal

a) The length of the school year in Egypt is 36 weeks and in Indonesia is 44 weeks. Find the ratio of the length of the school year duration in Indonesia compared to Egypt.

b) A computer screen with a diagonal of 24 inches has a length of 20 inches. Find the ratio of the length to the diagonal.

= = :

c) The platinum alloy commonly used in the USA is 90% platinum and 10% iridium. Find the ratio of iridium to platinum in the alloy.

iridium : platinum : ______ d) In 1978 only 8% of U.S. households had microwave ovens. As of 2006 over 80% have them. Find the ratio of microwave oven ownership in 2006 to 1978.

: = :

e) The 8-carat gold is 33% gold, 20% silver and the rest is copper. Find the ratio of silver to other components.

f) For children aged 2 to 11 years, an airfare is 75% of the full adult airfare. Find the ratio of child to adult airfares.

: : = = :

= = :

Skill 14.8 Finding the ratio of two quantities (2). The London Eye has 32 capsules and The Southern Star Observation Wheel (Melbourne) has a capacity of 20 the Singapore Flyer observation wheel has 28 capsules. Find the ratio of passengers per capsule and the London capsules in the Singapore Flyer to Eye has a capacity of 25. Find the ratio of the London Eye passengers per capsules in the London Eye. capsule to the Southern Star. In 2008, of the 100 seats in the Senate, Find the ratio of the height of the Statue i) 84 are held by men. What is the ratio of Liberty (150 ft) to the height of the of women to men in the Senate? Sears Tower, Chicago (1700 ft). k) A soccer field is 120 yards long and The lowest temperature recorded in the 80 yards wide. Find the ratio of width to USA is -80°F and in Russia is -90°F. Find the ratio of the lowest temperature length. in the USA compared to Russia. **m)** The lowest temperature recorded in **n)** The sensory, language and memory centers are located in the temporal lobe, North America is –81°F and in South America is -27° F. Find the ratio of the which is 22% of the total cerebral cortex lowest temperature in South America volume in the brain. Find the ratio of the compared to North America. temporal lobe to the rest of the cortex.

Skill 14.9 Finding other rate	Skill	14.9	Finding	other	rates
-------------------------------	-------	------	---------	-------	-------

$$rate = \frac{amount}{time}$$

Rate of change

Divide the amount by the time taken.

Example: A 300 L bathtub can be filled in 10 minutes.

Rate =
$$\frac{30\% L}{1\% min}$$
 = 30 L/min

amount = rate · time

Amount

 Multiply the rate by the time taken.

Example: Sam worked 7 h at a rate of \$16/h.

Amount (pay) =
$$16 \cdot 7$$

= \$112

b)

$$time = \frac{amount}{rate}$$

Time taken

Divide the amount by the rate.
 Example: A Lexmark E232
 prints 990 pages at a rate of 22 pages/min (ppm).

 $Time = \frac{990 \text{ p}}{22 \text{ ppm}} = 45 \text{ min}$

- Q. Some species of bamboo can grow 90 feet per year. At this rate how long will they grow in a month?
- **A.** rate = 90 ft per year 1 year = 12 months $rate \ per \ month = 90$ ft \div 12 = 7.5 ft
- a) A Mini Cooper Diesel with a 1.6 L engine emits 104 g/km of the greenhouse gas carbon dioxide (CO₂). How many grams of CO₂ will be emitted during a 400 km trip?

 $amount(g) = rate(g/km) \cdot distance(km)$

$$= 104 \text{ g/km} \cdot 400 \text{ km} =$$

41,600 g

amount = = = m

Most of the Lambert Glacier (Antarctica)

moves around 150 meters in 4 months.

At this rate how much will it move in

c) The Kudzu climbing plant can grow up to 364 ft per year. What is this rate in feet per week?

 $1 \ year = 52 \ weeks$

$$rate/wk = 364 ft \div 52 wk =$$

d) It takes 45 minutes to fill a 180-gallon swimming pool. What is the average rate in gallons per minute?

rate =

6 months?

e) A Dodge Dakota Pickup 2WD automatic has a city consumption of 15 L of gas per 100 km. How much gas does it need for a 20 km trip?

amount =

g) A Honda Civic Hybrid automatic has a highway consumption of 45 L of gas per 100 km. How much gas does it need for a 250 km trip?

amount =

f) Every glass bottle recycled saves enough energy to light a 100-watt light bulb for 4 hours. How many bottles are needed to light the same bulb for a week?

1 week =

h) The annual fuel cost for a Lamborghini Coupe is around \$2490. How much is the cost per month?

1 year =

