12. [Exploring Number]

Skill 12.1 Using 'order of operations' involving a mix of ( ), ×, ÷, + or –

- Follow this order of operations:
  Simplify inside the brackets.
  Multiply (×) and/or divide (÷) in order from left to right.
  Add (+) and/or subtract (–) in order from left to right.

Q. $18 \div (9 - 3) + 2 =$

A. $18 \div (9 - 3) + 2 =$
(simplify the brackets first)
$= 18 \div 6 + 2$
(division before addition)
$= 3 + 2$
$= \mathbf{5}$

a) $6 + 12 \div 4 \times 3 =$

$= 6 + 3 \times 3$

$= 6 + 9$ $=$ ☐

b) $6 \times 15 - 8 \times 3 =$

$=$ .....

$=$ ..... $=$ ☐

c) $5 + 12 \div 6 \times 3 =$

$=$ .....

$=$ ..... $=$ ☐

d) $3 \times (5 - 3) \times 8 =$

$=$ .....

$=$ ..... $=$ ☐

e) $(15 + 8) - (7 + 6) =$

$=$ .....

$=$ ..... $=$ ☐

f) $120 \div 5 - 6 \times 3 =$

$=$ .....

$=$ ..... $=$ ☐

g) $7 \times 8 + 24 \div 2 - 5 \times 6 =$

$=$ .....

$=$ ..... $=$ ☐

h) $120 \div (2 + 5 \times 2) - 9 =$

$=$ .....

$=$ ..... $=$ ☐

i) $100 - 5 \times (4 + 8) \div 4 - 55 =$

$=$ .....

$=$ ..... $=$ ☐

j) $13 + 6 \times 3 - 5 \times (2 + 4) =$

$=$ .....

$=$ ..... $=$ ☐

k) $27 - 3 \times (5 + 7) \div 6 + 10 =$

$=$ .....

$=$ ..... $=$ ☐

l) $5 + 3 \times 9 - 3 \times (2 + 8) =$

$=$ .....

$=$ ..... $=$ ☐

Skill 12.2 Using 'order of operations' involving powers and ( ), ×, ÷, + or –

- Follow this order of operations:
  Simplify inside the brackets.
  Evaluate all powers.
  Multiply (×) and/or divide (÷) in order from left to right.
  Add (+) and/or subtract (–) in order from left to right.

Q. $(6 + 2 \times 5)^2 =$

A. $(6 + 2 \times 5)^2 =$
multiply within brackets first
$= (6 + 10)^2$
add within the brackets
$= 16^2$
$= \mathbf{256}$

a) $(3 + 5)^2 =$

$= 8^2$ ..... $=$ ☐

b) $(12 - 7)^2 =$

$=$ ..... $=$ ☐

c) $(5 + 5 \times 3)^2 =$

$=$ .....

$=$ ..... $=$ ☐

d) $(2 \times 4 + 6)^2 =$

$=$ .....

$=$ ..... $=$ ☐

e) $(2 + 8)^2 \div 4 =$

$=$ .....

$=$ ..... $=$ ☐

f) $(7 + 5)^2 \div 8 =$

$=$ .....

$=$ ..... $=$ ☐

g) $5 + (12 - 6)^2 =$

$=$ .....

$=$ ..... $=$ ☐

h) $8 + (13 - 8)^2 =$

$=$ .....

$=$ ..... $=$ ☐

i) $(4 \times 2 + 2)^2 =$

$=$ .....

$=$ ..... $=$ ☐

j) $(3 \times 4 + 8)^2 =$

$=$ .....

$=$ ..... $=$ ☐

k) $(10 - 6)^2 \div (20 - 18) =$

$=$ .....

$=$ ..... $=$ ☐

l) $(10 - 5)^2 \div (14 - 9) =$

$=$ .....

$=$ ..... $=$ ☐

Skill 12.3 Rounding decimal numbers to a given place.

To round terminating decimals to a given place:

- Circle the digit to the right of the requested place.
- If this digit is: 0, 1, 2, 3 or 4 (< 5) - **round down** - keep the digit in the requested place the same.
  5, 6, 7, 8 or 9 (≥ 5) - **round up** - add 1 to the digit in the requested place.

To round repeating decimals to a given place:

- Write the first 4 digits after the decimal point. (see skill 7.7, page 81)
- Apply the procedure described above for terminating decimals.

Q. Round $0.4\overline{6}$ to the nearest thousandth.

A. $0.4\overline{6} = 0.4666...$ (6 is repeating indefinitely)

$0.4666..$ (circle the fourth digit)

$\approx \mathbf{0.467}$ ($6 \geq 5$, round up by adding 1 to 6)

a) Round 0.13 to the nearest tenth.

0.13 ($3 < 5$, round down by keeping 1) 0.1

b) Round 7.89 to the nearest tenth.

.....

c) Round 12.45 to the nearest tenth.

.....

d) Round 31.5841 to the nearest hundredth.

31.5841 ($4 < 5$, round down by keeping 8) 31.58

e) Round 24.793 to the nearest hundredth.

.....

f) Round 4.231 to the nearest hundredth.

.....

g) Round 3.859 to the nearest tenth.

.....

h) Round 50.296 to the nearest hundredth.

.....

i) Round $4.\overline{7}$ to the nearest hundredth.

$4.\overline{7} = 4.777...$ ($7 \geq 5$, round up by adding 1 to 7) $\approx$ 4.78

j) Round $3.\overline{42}$ to the nearest hundredth.

..... $\approx$

k) Round $0.\overline{6}$ to the nearest hundredth.

..... $\approx$

l) Round $1.7\overline{3}$ to the nearest thousandth.

..... $\approx$

m) Round $4.2\overline{8}$ to the nearest thousandth.

..... $\approx$

n) Round $0.\overline{16}$ to the nearest thousandth.

..... $\approx$

Skill 12.4 Writing rational approximations of simple irrational numbers.

irrational number $\sqrt{2} \approx 1.41421356$ rational approximation

- Circle the digit to the right of the requested place.
- If this digit is: 0, 1, 2, 3 or 4 (< 5) - **round down** - keep the digit in the requested place the same.
  5, 6, 7, 8 or 9 (≥ 5) - **round up** - add 1 to the digit in the requested place.

Hint: To write a decimal number correct to two decimal places is the same thing as rounding off to the the nearest hundredth.

Q. cos 45° ≈ 0.70711
Write the rational approximation of cos 45° correct to two decimal places.

A. *0.707*11
circle the third digit

0.71 *7 ≥ 5, round up by adding 1 to 0*

a) $\sqrt{12} \approx 3.46410162$
Write the rational approximation of $\sqrt{12}$ correct to two decimal places.

*3.464*10162 *4 < 5, round down by keeping 6* ≈ 3.46

b) $\sqrt{20} \approx 4.47213595$
Write the rational approximation of $\sqrt{20}$ correct to two decimal places.

..... ≈ .....

c) $\sqrt{24} \approx 4.89897949$
Write the rational approximation of $\sqrt{24}$ correct to two decimal places.

..... ≈ .....

d) $\sqrt{30} \approx 5.47722558$
Write the rational approximation of $\sqrt{30}$ correct to two decimal places.

..... ≈ .....

e) $\pi \approx 3.14159265$
Write the rational approximation of π correct to three decimal places.

..... ≈ .....

f) $\varphi \approx 1.61803398$
Write the rational approximation of φ correct to three decimal places.

..... ≈ .....

g) sin 15° ≈ 0.25882
Write the rational approximation of sin 15° correct to three decimal places.

..... ≈ .....

h) tan 60° ≈ 1.73205
Write the rational approximation of tan 60° correct to three decimal places.

..... ≈ .....

i) e ≈ 2.71828182 (Euler's number)
Write the rational approximation of e correct to two decimal places.

..... ≈ .....

j) $\sqrt{10} \approx 3.16227766$
Write the rational approximation of $\sqrt{10}$ correct to three decimal places.

..... ≈ .....

Skill 12.5 Writing very large and very small numbers in scientific notation.

243,000	=	2.43×10^5	0.000802	=	8.02×10^{-4}
Standard Form Very large		**Scientific Notation** Product of: Number ≥ 1 and < 10; Power of 10 with positive exponent	**Standard Form** Very small		**Scientific Notation** Product of: Number ≥ 1 and < 10; Power of 10 with negative exponent

To find the number greater than or equal to 1 and less than 10:
- Move the decimal point after the first non-zero digit and count the places moved.
- Write this number ignoring the zeros at either end.

To find the power of 10:
- Write base 10 with an exponent equal to the number of places moved.
- Check the sign of the exponent: Decimal point moves left ⇒ positive exponent.
  Decimal point moves right ⇒ negative exponent.

Q. Write 384,000,000 meters, the distance from earth to moon, in scientific notation.

A. $384,000,000$ (8 places)
$= 3.84 \times 100,000,000$ (8 zeros)
$= \mathbf{3.84 \times 10^8}$ (number ≥ 1 and < 10; exponent = +8 because of the 8 places and large original number (> 3.84))

a) 73,000 written in scientific notation is:
A) 7.3×10^{-4} B) 0.73×10^5 C) 7.3×10^4

$73,000$ (4 places; 4 zeros; exponent = +4)
$= 7.3 \times 10,000 = 7.3 \times 10^4$ = C

b) 5,010,000 written in scientific notation is:
A) 5.01×10^6 B) 5.1×10^5 C) 5.01×10^5

$5,010,000$
= =

c) Some people have as many as 150,000 hair follicles on their head. Write this in scientific notation.

=

d) By 2010 the world's population is estimated to reach 6.84 billion people. Write this in scientific notation.

=

e) 0.00304 written in scientific notation is:
A) 3.4×10^{-4} B) 3.04×10^{-3} C) 3.04×10^{-4}

0.00304 (3 places; 3 zeros; exponent = −3)
$= 3.04 \times 1000 = 3.04 \times 10^{-3}$ =

f) 0.000051 written in scientific notation is:
A) 5.1×10^{-5} B) 5.1×10^5 C) 5.1×10^{-4}

= =

g) Write 0.00000024 m, the diameter of a human hair strand, in scientific notation.

=

h) In diamonds, the distance between two carbon atoms is 0.000000142 m. Write this in scientific notation.

=

Skill 12.6 Expressing numbers in standard form.

2.43×10^5 = 243,000

Scientific Notation
Product of: Number ≥ 1 and < 10
Power of 10 with positive exponent

Standard Form
Very large

8.02×10^{-4} = 0.000802

Scientific Notation
Product of: Number ≥ 1 and < 10
Power of 10 with negative exponent

Standard Form
Very small

If the power of 10 is **positive:**

- Move the decimal point to the right as many places as the power of 10.
- Add zeros as place holders if necessary.
  Example: 3.1 = 3.1000
  Hint: By convention 37 = 37. = 37.0

If the power of 10 is **negative:**

- Move the decimal point to the left as many places as the power of 10.
- Add zeros as place holders if necessary.
  Example: 4.5 = 00004.5
- If the result is less than 1, write a zero in the units place.
  Hint: By convention 0.37 not .37

Q. Write 3.5×10^{-4} m, the diameter of optical fiber, in standard form.

A. 3.5×10^{-4} = (exponent = −4)
= 00003.5 × 10^{-4} (move decimal point 4 places left; add zeros as place holders)
= **0.00035**

a) 6.2×10^5 is the scientific notation for:
A) 6200 B) 620,000 C) 6.20000

(exponent = +5)
6.2×10^5 =
= 620,000.00 (5 places right) B

b) 4.12×10^6 is the scientific notation for:
A) 4,120,000 B) 412,000 C) 4.120000

=

c) Earth's atmosphere extends upward for 9.65×10^5 m. Write this in standard form.

d) Write 1.3×10^9, China's population in 2006, in standard form.

e) The size of a red blood cell, 8.0×10^{-3} mm, is scientific notation for:
A) 0.0008 B) 8000 C) 0.008

=

f) The size of a virus, 2.5×10^{-5} mm, is scientific notation for:
A) 0.00025 B) 0.000025 C) 250,000

=

g) Write 2.5×10^{-11} m, the radius of a hydrogen atom, in standard form.

h) Write 5×10^{-7} m, the size of a speck of dust, in standard form.

Skill 12.7 Using 'order of operations' involving negative integers.

- Follow this order of operations:
  Simplify inside the brackets.
  Multiply (×) and/or divide (÷) in order from left to right.
  Add (+) and/or subtract (–) in order from left to right.

Q. $-9 \div 3 - 5 \times (-10) =$

A. $-9 \div 3 - 5 \times (-10) =$
$= -3 - (-50)$ *divide and multiply first*
$= -3 + 50$
$= 47$ *subtract, use "+"*

a) $(12 - 2) \div (9 - 11) =$
$= 10 \div (-2)$ $= -5$

b) $-56 \div 8 - 4 \times 7 =$
$=$ ..... $=$

c) $-12 \div 4 + (-3) \times 4 =$
$=$ ..... $=$

d) $(-15 + 11) \times (9 - 14) =$
$=$ ..... $=$

e) $-8 \times 6 - 42 \div 7 =$
$=$ ..... $=$

f) $(9 - 17) \div (-6 + 8) =$
$=$ ..... $=$

g) $(13 + 11) \times (7 - 10) =$
$=$ ..... $=$

h) $-3 \times 7 - 12 \div 6 =$
$=$ ..... $=$

i) $15 - 8 \times (10 - 5) \div 4 - 12 =$
$=$ ..... $=$

j) $-7 + 12 \div (4 + 8) \times 6 + 10 =$
$=$ ..... $=$

k) $-4 + 2 \times (8 - 12) \div 4 - 12 =$
$=$ ..... $=$

l) $1 - 16 \div (3 + 5) \times 3 - 15 =$
$=$ ..... $=$

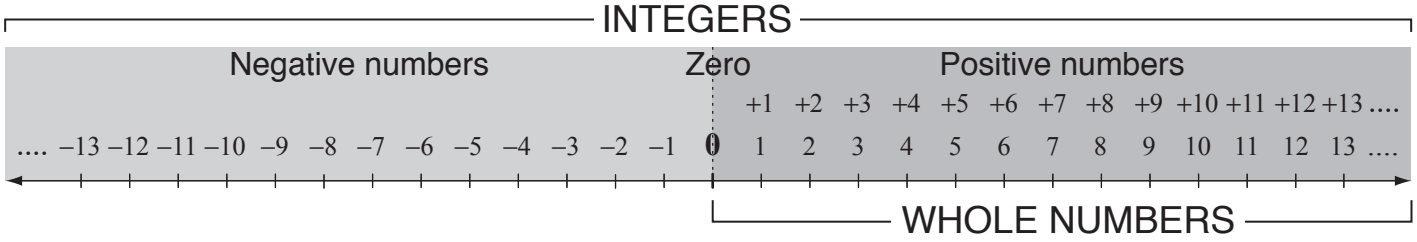
m) $-10 \times (14 - 7) =$
$=$ ..... $=$

n) $-28 \div (-1 - 6) + 17 =$
$=$ ..... $=$

o) $-9 - 4 \times (13 - 2) =$
$=$ ..... $=$

p) $-9 - 2 \times (19 - 3) =$
$=$ ..... $=$

Skill 12.8 Recognizing whole numbers and integers.



- Decide if a number is a whole number or an integer, based on their definition and hints below. (see Glossary)

Hints: Negative numbers, fractions, terminating decimals, repeating decimals and infinite non-repeating decimals are not whole numbers.

Any fraction whose numerator is divisible by the denominator is a whole number: $\frac{12}{4} = 3$

Any decimal with only zeros after the decimal point is a whole number: $8.00 = 8$

Fractions, terminating decimals, repeating decimals and infinite non-repeating decimals are not integers.

Any fraction whose numerator is divisible by the denominator is an integer: $-\frac{5}{1} = -5$, $\frac{12}{4} = 3$

Any decimal with only zeros after the decimal point is an integer: $8.00 = 8$

Any square root of a perfect square is an integer: $\sqrt{16} = 4$

Q. Choose the whole numbers from this list:
$-7, \frac{8}{2}, -\frac{1}{3}, 0, -3.6, 50$

A. *-7 is negative, so not a whole number*
$\frac{8}{2} = 8 \div 2 = 4$ is a whole number
$-\frac{1}{3}$ is a fraction, so not a whole number
-3.6 is a decimal, so not a whole number
So $\frac{8}{2}$***, 0, 50*** *are whole numbers.*

a) Choose the whole numbers from this list:
$7.43, (89), -5, 3\frac{1}{5}, (14), 0.6$

b) Choose the whole numbers from this list:
$567, 0.73, -4, \frac{3}{10}, 12, 0$

c) Choose the whole numbers from this list:
$1.4142, 18, -5.\overline{9}, \frac{4}{11}, -5, 143$

d) Choose the whole numbers from this list:
$-25, 0.6666..., 34, \frac{5}{7}, -1, 8.93567$

e) Choose the integers from this list:
$-3.5, 11, 2.\overline{14}, -1, 3\frac{2}{7}, 2$

f) Choose the integers from this list:
$3.14, \frac{16}{4}, -3, -0.\overline{72}, \sqrt{25}$

g) Choose the integers from this list:
$-75, 2.23607, -\frac{8}{2}, \sqrt{90}, 10.00$

h) Choose the integers from this list:
$-\sqrt{4}, \frac{\pi}{4}, 0.5252, 18, 0$

Skill 12.9 Recognizing rational numbers.

A number is **rational** if:

- It can be written as a fraction (ratio) of two integers.

Hint: *All integers are rational numbers:* $-2,\ 700,\ \sqrt{16},\ \frac{5}{1},\ \frac{25}{5}$

All terminating decimals are rational numbers: $2.16,\ -5.753469$

All repeating decimals are rational numbers: $0.57575757... = 0.\overline{57}$

Q. Which numbers are rational?

A) $-\sqrt{\frac{3}{5}}$

B) 0.999...

C) 0.12357102...

D) $\frac{11}{2}$

A. $-\sqrt{\frac{3}{5}}$ *is not rational, because* $\frac{3}{5}$ *is not a perfect square.*

0.999... is rational, because it is a repeating decimal.

0.12357102... is not rational, because it has infinite non-repeating digits after the decimal point.

$\frac{11}{2}$ *is rational, because it is a fraction.*

So ***B and D*** *are rational.*

a) Choose the rational numbers from this list:

$1.41421356...,\ \sqrt{\frac{1}{4}},\ -\frac{1}{3},\ 2.18,\ \frac{\pi}{6}$

b) Choose the rational numbers from this list:

$-5.\overline{2},\ \frac{3\pi}{2},\ \sqrt{10},\ 3.14,\ \frac{659}{3867}$

c) Which numbers are rational?

A) $\sqrt{\frac{4}{9}}$ B) $-\frac{\pi}{4}$

C) 3.14159265... D) 1.75

A and D

d) Which numbers are **not** rational?

A) $3.4\overline{8}$ B) 1.61803399...

C) $\frac{\pi}{2}$ D) $\sqrt{16}$

e) Which numbers are rational?

A) $\frac{21}{55}$ B) 0.00007

C) 6.9205729744... D) $-\sqrt{6}$

f) Which numbers are rational?

A) $\sqrt{10}$ B) $77.\overline{7}$

C) $-\frac{11}{2}$ D) $\sqrt{\frac{3}{7}}$

g) Which numbers are rational?

A) $\sqrt{8}$ B) $6.5\overline{9}$

C) –4.131133111333... D) 3.161616...

h) Which numbers are **not** rational?

A) –0.315315315... B) $\sqrt{3}$

C) 2.135791113... D) $\frac{11}{49}$

Skill 12.10 Recognizing irrational numbers.

A number is **irrational** (not rational) if:

- It can be written as a decimal, but not as a fraction.
- It has infinite non-repeating digits after the decimal point: 2.52849302953...

Hint: Square roots of prime numbers or rational numbers that are not perfect squares are irrational numbers: $\sqrt{5}$, $\sqrt{18}$
Special numbers, such as π, e, φ are irrational.
Some values of trigonometric and logarithmic functions are irrational.

Q. Which is an irrational number?

A) $\sqrt{900}$ B) $\frac{1}{20}$
C) $-\sqrt{38}$ D) -5.75

A. *$\sqrt{900}$ is rational, because it equals 30.*
$\frac{1}{20}$ is rational, because it is a fraction.
$\sqrt{38}$ is irrational, because it is a square root of a rational number that is not a perfect square.
5.75 is rational, because it is a decimal.

*So **C** is an irrational number.*

a) Which is an irrational number?

A) φ B) 120
C) $\sqrt{25}$ D) -0.1675

A

b) Which is an irrational number?

A) 0 B) $-5.636363...$
C) $\frac{3}{17}$ D) $\frac{1}{\sqrt{2}}$

c) Which is an irrational number?

A) 3 B) -2.5
C) $\sqrt{2}$ D) $-\sqrt{4}$

d) Which is an irrational number?

A) $2.\overline{6}$ B) 6.15
C) $\sqrt{7}$ D) $5\frac{3}{10}$

e) Which is an irrational number?

A) $\frac{659}{3867}$ B) 2.7182813...
C) $-9.\overline{42}$ D) $\sqrt{\frac{9}{4}}$

f) Which is an irrational number?

A) 3.15315315.... B) 5001
C) $\frac{3}{10}$ D) $\sqrt{18}$

g) Choose the irrational numbers from this list:

$\sqrt{15}, -\frac{2}{3}, -6, 0.13133133313..., \sqrt{5}$

h) Choose the irrational numbers from this list:

$-\frac{3}{14}, \sqrt{\frac{3}{5}}, \pi, \frac{53}{83}, 0.12357102...$

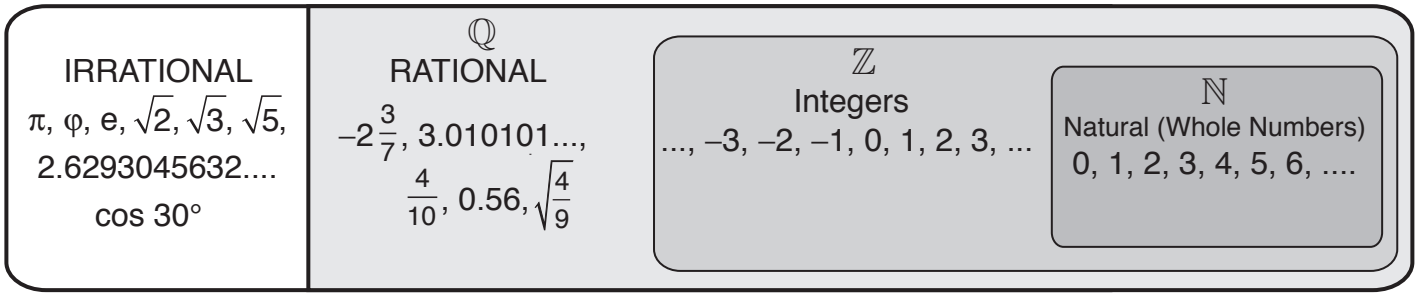
i) Choose the irrational numbers from this list:

$\sqrt{30}, 0.\overline{67}, 6.921921921..., -\sqrt{3}, \frac{\pi}{2}$

j) Choose the irrational numbers from this list:

$1\frac{1}{17}, -45, 3.14159..., \sqrt{\frac{6}{7}}, \varphi$

Skill 12.11 Recognizing classes of numbers.



Hint: *Rational numbers include integers, terminating decimals and repeating decimals.*
Irrational numbers include infinite non-repeating decimals.

$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$

Irrational numbers $\subset \mathbb{R}$

Q. Which classes of numbers describe $-\sqrt{81}$?

A) integer and irrational
B) rational and real
C) irrational and rational
D) real and natural

A. $-\sqrt{81} = -9$

integer	✔
rational	✔
real number	✔
natural	✘
irrational	✘

*So **B** is the correct description.*

a) Use true and false to complete this table:

	integer	rational	irrational	real
4.327	false	true	false	true

b) Use true and false to complete this table:

	integer	rational	irrational	real
–500				

c) Use true and false to complete this table:

	integer	rational	irrational	real
π				

d) Use true and false to complete this table:

	integer	rational	irrational	real
$\frac{3}{14}$				

e) Which classes of numbers describe 0.65291...?

A) integer and rational
B) rational and real
C) integer and irrational
D) irrational and real

f) Which classes of numbers describe $-\sqrt{49}$?

A) integer and rational
B) irrational and real
C) integer and irrational
D) rational and irrational

g) Which classes of numbers describe $0.\overline{1}53846\overline{}$?

A) integer and irrational
B) irrational and real
C) integer and rational
D) rational and real

h) Which classes of numbers describe $\frac{257}{43}$?

A) integer and rational
B) irrational and real
C) rational and real
D) rational and irrational

Skill 12.12 Comparing and ordering real numbers.

- Simplify the radical to the simplest form.

To estimate the value of a radical:
EITHER
- Find the perfect squares greater than (>) and less than (<) the number inside the square root.

OR
- Find the rational approximation of the radical from a table of values. (see Math Facts, page 453)
- Express the real numbers as decimals. (see skill 7.6, page 80)
- Order the decimal numbers. (see skill 7.1, page 75)

Q. Place in descending order:
$\sqrt{7}, \frac{7}{3}, 2.3, \frac{10}{4}, 2.41$

A. $\sqrt{7} = 2.6457...$
$\frac{7}{3} = 2.3333...$
$\frac{10}{4} = 2.5$
Descending means from the largest to smallest: 2.6457, 2.5, 2.41, 2.3333, 2.3
The order is: $\sqrt{7}, \frac{10}{4}, 2.41, \frac{7}{3}, 2.3$

a) Which number is greater:
π or 3.1? ($\pi \approx 3.14159$)

π

b) Which number is greater:
π or $\sqrt{10}$?

c) Which number is smaller:
$\sqrt{20}$ or 5? ($\sqrt{20} = 2\sqrt{5} \approx 4.4721$)

d) Which number is smaller:
$\sqrt{6}$ or 2?

e) Which number is smaller:
5 or $\sqrt{32}$?

f) Which number is greater:
$\sqrt{50}$ or 7?

g) Place in ascending order:
$\sqrt{6}, \frac{3}{2}, 2.6, 2.\overline{4}$

h) Place in descending order:
$\sqrt{5}, \frac{5}{2}, 1.2, \sqrt{2}, 1.5$

i) Place in descending order:
$\frac{8}{3}, \sqrt{4}, \sqrt{8}, 2.6, 2.75$

j) Place in ascending order:
$3.2, \sqrt{9}, \pi, \frac{9}{4}, \sqrt{7}$