13. [Number Patterns]

Skill 13.1 Completing number patterns by adding, subtracting, multiplying or dividing by the same positive integer.

MMMauve 1 1 2 2 3 3 4 4
MMLime 1 1 2 2 3 3 4 4

- Look at consecutive terms of the pattern.
- Find the operation used to get from one term to the next.
  Hint: Every number pattern is created by adding, subtracting, multiplying or dividing by the same positive integer.
- Define the rule (operation) of the pattern.
- Apply this rule to the last given term and find the next two terms of the pattern.

Q. Complete the pattern:
135 , 120 , 105 , 90 , 75 , ____ , ____

A. *135 , 120 , 105 , 90 , 75 , ____ , ____*
−15 −15 −15 −15

Rule: *Subtract 15 from each term.*
The pattern is formed by subtracting 15.
75 − 15 = 60
60 − 15 = 45
*135 , 120 , 105 , 90 , 75 , **60** , **45***

a) What is the value of the missing term in the pattern?

term number	1	2	3	4	5	6
term value	1	9	17	25	33	?

+8 +8 +8 +8 +8

25 + 8 = 33 33 + 8 = 41

b) What is the value of the missing term in the pattern?

term number	1	2	3	4	5	6
term value	15	11	7	3	−1	?

c) What is the value of the missing term in the pattern?

term number	1	2	3	4	5	6
term value	2	10	50	250	1250	?

d) What is the value of the missing term in the pattern?

term number	1	2	3	4	5	6
term value	57	48	39	30	21	?

e) Complete the pattern:
3 , 11 , 19 , 27 , 35 , ____ , ____
+8 +8 +8 +8 +8 +8

f) Complete the pattern:
1215 , 405 , 135 , 45 , ____ , ____
÷3

g) Complete the pattern:
1 , 2 , 4 , 8 , 16 , ____ , ____

h) Complete the pattern:
3 , 6 , 12 , 24 , 48 , ____ , ____

Skill 13.2 Completing number patterns by adding or subtracting variable numbers.

- Look at consecutive terms of the pattern.
- Find the operation used to get from one term to the next.
  Hint: Every number pattern is created by adding or subtracting integers following a certain pattern.
- Define the rule (operation) of the pattern.
- Apply this rule to the last given term and find the next two terms of the pattern.
  Hint: Counting numbers, even numbers and odd numbers have patterns themselves that can become part of the rule (see below).

Q. Complete the pattern:
3 , 5 , 9 , 15 , 23 , ____ , ____

A. *3 , 5 , 9 , 15 , 23 , ____ , ____*
+2 +4 +6 +8

Rule: *Add 2, then 4, then 6, then 8, etc.*
The pattern is formed by adding consecutive even numbers.
23 + 10 = 33
33 + 12 = 45
*3 , 5 , 9 , 15 , 23 , **33** , **45***

a) Complete the pattern:
11 , 13 , 16 , 20 , 25 , ____ , ____
+2 +3 +4 +5 +6 +7
25 + 6 = 31 31 + 7 = 38

b) Complete the pattern:
2 , 3 , 5 , 8 , 12 , ____ , ____

c) Complete the pattern:
2 , 6 , 14 , 26 , 42 , ____ , ____

d) Complete the pattern:
1 , 3 , 7 , 13 , 21 , ____ , ____

e) Complete the pattern:
1 , 4 , 10 , 19 , 31 , ____ , ____

f) Complete the pattern:
5 , 6 , 9 , 14 , 21 , ____ , ____

g) Complete the pattern:
51 , 50 , 47 , 42 , 35 , ____ , ____

h) Complete the pattern:
53 , 50 , 45 , 38 , 29 , ____ , ____

i) Complete the pattern:
10 , 8 , 5 , 1 , –4 , ____ , ____

j) Complete the pattern:
16 , 14 , 10 , 4 , –4 , ____ , ____

Skill 13.3 Completing number patterns by adding or subtracting the same positive number to integers.

- Look at consecutive terms of the pattern.
- Find the operation used to get from one term to the next.
  Hint: Every number pattern is created by adding or subtracting the same positive integer.
- Define the rule (operation) of the pattern.
- Apply this rule to the last given term and find the next two terms of the pattern.

Q. Complete the pattern:
21 , 13 , 5 , –3 , –11 , ___ , ___

A. *21 , 13 , 5 , –3 , –11 , ___ , ___*
–8 –8 –8 –8
Rule: Subtract 8 from each term.
–11 – 8 = –19
–19 – 8 = –27
*21 , 13 , 5 , –3 , –11 , **–19** , **–27***

a) Complete the pattern:
8 , 5 , 2 , –1 , –4 , –7 , –10
–3 –3 –3 –3 –3 –3
–4 – 3 = –7 *–7 – 3 = –10*

b) Complete the pattern:
–16 , –11 , –6 , –1 , 4 , ___ , ___
+5

c) Complete the pattern:
–20 , –14 , –8 , –2 , 4 , ___ , ___

d) Complete the pattern:
9 , 5 , 1 , –3 , –7 , ___ , ___

e) Complete the pattern:
10 , 7 , 4 , 1 , –2 , ___ , ___

f) Complete the pattern:
–35 , –28 , –21 , –14 , –7 , ___ , ___

g) Complete the pattern:
–19 , –15 , –11 , –7 , –3 , ___ , ___

h) Complete the pattern:
30 , 22 , 14 , 6 , –2 , ___ , ___

i) Complete the pattern:
–21 , –12 , –3 , 6 , 15 , ___ , ___

j) Complete the pattern:
16 , 10 , 4 , –2 , –8 , ___ , ___

k) Complete the pattern:
12 , 7 , 2 , –3 , –8 , ___ , ___

l) Complete the pattern:
46 , 34 , 22 , 10 , –2 , ___ , ___

Skill 13.4 Completing number patterns by multiplying or dividing by the same integer.

- Look at consecutive terms of the pattern.
- Find the operation used to get from one term to the next.
  Hint: Every number pattern is created by multiplying or dividing by the same integer.
- Define the rule (operation) of the pattern.
- Apply this rule to the last given term and find the next two terms of the pattern.

Q. Complete the pattern:

$\frac{3}{49}$, $\frac{3}{7}$, 3 , 21 , 147 , ____ , ____

A. $\frac{3}{49}$, $\frac{3}{7}$, 3 , 21 , 147 , ____ , ____

$\times 7$ $\times 7$ $\times 7$ $\times 7$

Rule: *Multiply each term by 7.*

$147 \times 7 = 1029$

$1029 \times 7 = 7203$

$\frac{3}{49}$, $\frac{3}{7}$, 3 , 21 , 147 , ***1029*** , ***7203***

a) Complete the pattern:

288 , –144 , 72 , –36 , 18 , ____ , ____

$\div(-2)$ $\div(-2)$ $\div(-2)$ $\div(-2)$ $\div(-2)$ $\div(-2)$

$18 \div (-2) = -9 \quad -9 \div (-2) = 4.5$

b) Complete the pattern:

2 , –6 , 18 , –54 , 162 , ____ , ____

$\times(-3)$

c) Complete the pattern:

–1 , 5 , –25 , 125 , ____ , ____

d) Complete the pattern:

–200,000 , 20,000 , –2000 , 200 , ____ , ____

e) Complete the pattern:

$\frac{1}{6}$, 1 , 6 , 18 , ____ , ____

f) Complete the pattern:

$\frac{3}{5}$, 3 , 15 , 75 , ____ , ____

g) Complete the pattern:

$\frac{7}{4}$, $\frac{7}{2}$, 7 , 14 , 28 , ____ , ____

h) Complete the pattern:

$\frac{1}{64}$, $\frac{1}{8}$, 1 , 8 , 64 , ____ , ____

i) Complete the pattern:

$\frac{2}{81}$, $\frac{2}{9}$, 2 , 18 , 162 , ____ , ____

j) Complete the pattern:

$\frac{11}{100}$, $\frac{11}{10}$, 11 , 110 , ____ , ____

Skill 13.5 Completing number patterns involving decimals and fractions.

- Look at consecutive terms of the pattern.
- Find the operation used to get from one term to the next.
  Hint: Every number pattern is created by adding, subtracting, multiplying or dividing by rational numbers (whole numbers, fractions or decimals).
- Define the rule (operation) of the pattern.
- Apply this rule to the last given term and find the next two terms of the pattern.

Q. Complete the pattern:

$\frac{5}{18}, \frac{1}{2}, \frac{13}{18}, \frac{17}{18}, 1\frac{1}{6}, \_\_\_\_, \_\_\_\_$

A. $\frac{5}{18}, \frac{1}{2}, \frac{13}{18}, \frac{17}{18}, 1\frac{1}{6}, \_\_\_\_, \_\_\_\_$

Look at the 3rd and 4th terms: their difference is $\frac{4}{18}$

Rule: *Add* $\frac{4}{18}$ *to each term.*

$1\frac{1}{6}+\frac{4}{18}=\frac{7}{6}+\frac{4}{18}=\frac{21}{18}+\frac{4}{18}=\frac{25}{18}=1\frac{7}{18}$

$\frac{25}{18}+\frac{4}{18}=\frac{29}{18}=1\frac{11}{18}$

$\frac{5}{18}, \frac{1}{2}, \frac{13}{18}, \frac{17}{18}, 1\frac{1}{6}, \mathbf{1\frac{7}{18}}, \mathbf{1\frac{11}{18}}$

a) Complete the pattern:

$1, 2, 3.5, 5.5, 8, \_\_\_\_, \_\_\_\_$

+1, +1.5, +2, +2.5, +3, +3.5

$8 + 3 = 11 \qquad 11 + 3.5 = 14.5$

b) Complete the pattern:

$0.8, 2, 3.4, 5, 6.8, \_\_\_\_, \_\_\_\_$

+1.2

c) Complete the pattern:

$1.5, 3.5, 6, 9, 12.5, \_\_\_\_, \_\_\_\_$

d) Complete the pattern:

$4, 5.5, 7.5, 10, 13, \_\_\_\_, \_\_\_\_$

e) Complete the pattern:

$1.75, 3.5, 7, 14, \_\_\_\_, \_\_\_\_$

f) Complete the pattern:

$1\frac{1}{4}, 2\frac{1}{2}, 5, 10, \_\_\_\_, \_\_\_\_$

g) Complete the pattern:

$36, 18, 9, \frac{9}{2}, \frac{9}{4}, \_\_\_\_, \_\_\_\_$

h) Complete the pattern:

$32, 8, 2, \frac{1}{2}, \frac{1}{8}, \_\_\_\_, \_\_\_\_$

i) Complete the pattern:

$3\frac{1}{4}, 4, 4\frac{3}{4}, 5\frac{1}{2}, 6\frac{1}{4}, \_\_\_\_, \_\_\_\_$

j) Complete the pattern:

$\frac{8}{15}, \frac{4}{5}, 1\frac{1}{15}, 1\frac{1}{3}, 1\frac{3}{5}, \_\_\_\_, \_\_\_\_$

Skill 13.6 Finding a term in a number pattern.

- Draw a table and list the given terms and also the position each term occupies in the pattern.
- Look for a relationship between consecutive terms and/or between the term and its position in the pattern.
- Based on this relationship, find the requested term in the pattern.

Q. Find the 30th term in the pattern
2 , 4 , 6 , 8 , 10 , ...

A.

position	1st	2nd	3rd	4th	5th	.....	30th
term	2	4	6	8	10		?
relationship	1 × 2	2 × 2	3 × 2	4 × 2	5 × 2		30 × 2

×2

Relationship: each term = twice its position.
The 30th term of the pattern is 2 × 30 = ***60***

a) Find the 20th term in the pattern
5 , 7 , 9 , 11 , 13 , ...

position	1st	2nd	3rd	4th	5th	.....	20th
term	5	7	9	11	13		?
relationship	1×2+3	2×2+3	3×2+3	4×2+3	5×2+3		20×2+3

×2+3

Relationship: term = position × 2 + 3

20th term = 20 × 2 + 3 = ☐

b) Find the 15th term in the pattern
5 , 10 , 15 , 20 , 25 , ...

position	1st	2nd	3rd	4th	5th	.....	15th
term	5	10	15	20	25		?
relationship	1 × 5	2 × 5	3 × 5	4 × 5	5 × 5		

?

Relationship: term =

15th term = = ☐

c) Find the 20th term in the pattern
8 , 13 , 18 , 23 , 28 , ...

position	1st	2nd	3rd	4th	5th	.....	20th
term	8	13	18	23	28		?
relationship	1×5+3	2×5+3	3×5+3	4×5+3	5×5+3		

?

Relationship: term =

20th term = = ☐

d) Find the 25th term in the pattern
4 , 6 , 8 , 10 , 12 , ...

position	1st	2nd	3rd	4th	5th	.....	25th
term	4	6	8	10	12		?
relationship							

?

Relationship: term =

25th term = = ☐

e) Find the 20th term in the pattern
1 , 4 , 7 , 10 , 13 , ...

position	1st	2nd	3rd	4th	5th	.....	20th
term	1	4	7	10	13		?
relationship							

?

Relationship: term =

20th term = = ☐

f) Find the 8th term in the pattern
1 , 2 , 4 , 8 , 16 , ...

position	1st	2nd	3rd	4th	5th	.....	8th
term	1	2	4	8	16		?
relationship							

?

Relationship: term =

8th term = = ☐

Skill 13.7 Finding a particular term of a sequence given its general rule.

- Identify the value of n for the requested term of the sequence.
  Hint: If t_{40} needs to be found, the value of n is 40.
- Substitute the value of n in the formula for the general rule of the pattern t_n
- Calculate the value of the particular term of the sequence.

Q. If the rule of a pattern is $t_n = \frac{n}{4} - 9$
find t_{60}

A. *t_{60} is the 60th term of the sequence.*

$t_n = \frac{n}{4} - 9$ ← *substitute n = 60*

$t_{60} = \frac{60}{4} - 9$

$= 15 - 9$

$= \mathbf{6}$

a) If the general rule of a sequence is $t_n = 5n - 4$
find t_{30}

$t_{30} = 5 \times 30 - 4$ ← *n = 30*

$= 150 - 4$ $=$ ☐

b) If the general rule of a sequence is $t_n = 4n - 7$
find t_{15}

$t_{15} =$

$=$ $=$ ☐

c) If the general rule of a sequence is $t_n = 8 - 5n$,
find t_{10}

$t_{10} =$

$=$ $=$ ☐

d) If the general rule of a sequence is $t_n = 25 - n$
find t_{40}

$t_{40} =$

$=$ $=$ ☐

e) If the general rule of a sequence is $t_n = 15n$
find t_{30}

$t_{30} =$

$=$ $=$ ☐

f) If the general rule of a sequence is $t_n = 40n$
find t_{25}

$t_{25} =$

$=$ $=$ ☐

g) If the general rule of a sequence is $t_n = -\frac{2n}{7}$
find t_{35}

$t_{35} =$

$=$ $=$ ☐

h) If the general rule of a sequence is $t_n = \frac{n}{3} + 1$
find t_{21}

$t_{21} =$

$=$ $=$ ☐

i) If the general rule of a sequence is $t_n = -6(n - 3)$
find t_{23}

$t_{23} =$

$=$ $=$ ☐

j) If the general rule of a sequence is $t_n = 3(n - 6)$
find t_{24}

$t_{24} =$

$=$ $=$ ☐

Skill 13.8 Finding two or more terms of a sequence given its general rule.

- Identify the values of n for the requested terms of the sequence.
  Hint: If $n \geq 3$ and the first 3 terms need to be found, the values of n are 3, 4 and 5.
- Substitute the values of n in the formula for the general rule of the sequence t_n
- Calculate the values of the particular terms of the sequence.

Q. Write the first four terms of the sequence where $t_n = 4 - n^2$ and $n \geq 2$

A. *$n \geq 2$ means that the first four terms are t_2, t_3, t_4 and t_5*

$n = 2 \Rightarrow t_2 = 4 - 2^2 = 4 - 4 = 0$

$n = 3 \Rightarrow t_3 = 4 - 3^2 = 4 - 9 = -5$

$n = 4 \Rightarrow t_4 = 4 - 4^2 = 4 - 16 = -12$

$n = 5 \Rightarrow t_5 = 4 - 5^2 = 4 - 25 = -21$

The first 4 terms are: ***0, −5, −12, −21***

a) Write the first three terms of the sequence where $t_n = 20 - 7n$ and $n \geq 1$

$n = 1 \Rightarrow t_1 = 20 - 7 \cdot 1 = 20 - 7 = 13$

$n = 2 \Rightarrow t_2 =$

$n = 3 \Rightarrow t_3 =$

b) Write the first three terms of the sequence where $t_n = n^2 - 6$ and $n \geq 2$

$n = 2 \Rightarrow t_2 =$

$n = 3 \Rightarrow t_3 =$

$n = 4 \Rightarrow t_4 =$

c) Write the first four terms of the sequence where $t_n = (-2)^n$ and $n \geq 1$

$n = 1 \Rightarrow t_1 =$

$n = 2 \Rightarrow t_2 =$

d) Write the first four terms of the sequence where $t_n = 9 - n^2$ and $n \geq 3$

$n = 3 \Rightarrow t_3 =$

$n = 4 \Rightarrow$

e) Write the first four terms of the sequence where $t_n = -\frac{3n}{2}$ and $n \geq 4$

$n = 4 \Rightarrow$

f) Write the first four terms of the sequence where $t_n = \frac{5n}{6}$ and $n \geq 3$

$n =$

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Skill 13.9 Finding the general rule of a sequence t_n (1).

To decide which general formula is true for the values shown in the table:

- Substitute the values for n (first row in the table) in the general formula.
- Check if the results match the term values (t_n) given in the second row of the table.

To express t_n in terms of n:

EITHER

- Look for a relationship between consecutive terms and/or between the term and its position in the sequence.

Hints: This relationship is an expression of n involving one or more operations, i.e. $t_n = n - 6$, $2n$, $4n + 2$

OR

- Find the difference between consecutive terms of the sequence (common difference).
- Write the term "common difference · n" in the expression.
- Check the result by substituting a random value for n into the formula.
- Adjust the expression by adding, subtracting, multiplying or dividing by a constant.
- Check the result by substituting all values for $n = 1, 2, 3, 4, 5$ into the final formula.

Q. Express t_n in terms of n given the table of values for the sequence.

number *(n)*	1	2	3	4	5	....	*n*
term t_n	1	5	9	13	17	....	

+4 +4 +4 +4 +4

A. *common difference = 4*

$\Rightarrow t_n$ contains 4n

If $t_n = 4n$ and $n = 1 \Rightarrow t_1 = 4 \cdot 1 = 4 \neq 1$

If $t_n = 4n - 3$ — Adjust by subtracting 3

$n = 1 \Rightarrow t_1 = 4 \cdot 1 - 3 = 1$ (true)

$n = 2 \Rightarrow t_2 = 4 \cdot 2 - 3 = 5$ (true)

$n = 3 \Rightarrow t_3 = 4 \cdot 3 - 3 = 9$ (true)

$n = 4 \Rightarrow t_4 = 4 \cdot 4 - 3 = 13$ (true)

$n = 5 \Rightarrow t_5 = 4 \cdot 5 - 3 = 17$ (true)

The general term is $\boldsymbol{t_n = 4n - 3}$

a) The rule for the general term of this sequence is:

A) $5n + 2$ B) $2n - 5$

C) $5n + 5$ D) $2n + 5$

number *(n)*	1	2	3	4	5	6	....
term t_n	7	9	11	13	15	17	....

A $n = 1 \Rightarrow t_1 = 5 \cdot 1 + 2 = 7 \Rightarrow$ *true*

$n = 2 \Rightarrow t_2 = 5 \cdot 2 + 2 = 12 \Rightarrow$ *false*

B $n = 1 \Rightarrow t_1 = 2 \cdot 1 - 5 = -3 \Rightarrow$ *false*

C $n = 1 \Rightarrow t_1 = 5 \cdot 1 + 5 = 10 \Rightarrow$ *false*

D $n = 1 \Rightarrow t_1 = 2 \cdot 1 + 5 = 7 \Rightarrow$ *true*

$n = 2 \Rightarrow t_2 = 2 \cdot 2 + 5 = 9 \Rightarrow$ *true*

$n = 3 \Rightarrow t_3 = 2 \cdot 3 + 5 = 11 \Rightarrow$ *true*

$n = 4 \Rightarrow t_4 = 2 \cdot 4 + 5 = 13$ $\Rightarrow$ D

b) The rule for the general term t_n of this pattern is:

A) $3n - 4$ B) $3n + 4$

C) $4n + 3$ D) $4n - 3$

number *(n)*	1	2	3	4	5	6	....
term t_n	7	10	13	16	19	22	....

A

$\Rightarrow$

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Skill 13.9 Finding the general rule of a sequence t_n (2).

c) Express t_n in terms of n given the table of values for the sequence.

number *(n)*	1	2	3	4	5	....	n
term t_n	8	10	12	14	16	....	$2n + 6$

common difference = 2

$t_n = 2n \Rightarrow t_1 = 2 \cdot 1 = 2$ *(false)*

adjust $t_n = 2n + 6 \Rightarrow t_3 = 2 \cdot 3 + 6 = 12$ *(true)*

d) Express t_n in terms of n given the table of values for the sequence.

number *(n)*	1	2	3	4	5	....	n
term t_n	5	4	3	2	1	....	

common difference = 1

$t_n = n \Rightarrow t_1 = 1$ *(false)*

adjust $t_n = 6 - n \Rightarrow t_4 = 6 - 4 = 2$ *(true)*

e) Express t_n in terms of n given the table of values for the sequence.

number *(n)*	1	2	3	4	5	....	n
term t_n	2	5	8	11	14	....	

common difference =

$t_n =$

adjust $t_n =$

f) Express t_n in terms of n given the table of values for the sequence.

number *(n)*	1	2	3	4	5	....	n
term t_n	5	9	13	17	21	....	

common difference =

$t_n =$

adjust $t_n =$

g) Express t_n in terms of n given the table of values for the sequence.

number *(n)*	1	2	3	4	5	....	n
term t_n	5	10	15	20	25	....	

common difference =

$t_n =$

h) Express t_n in terms of n given the table of values for the sequence.

number *(n)*	1	2	3	4	5	....	n
term t_n	8	11	14	17	20	....	

common difference =

$t_n =$

i) Express t_n in terms of n given the table of values for the sequence.

number *(n)*	1	2	3	4	5	....	n
term t_n	2	1	0	–1	–2	....	

common difference =

$t_n =$

j) Express t_n in terms of n given the table of values for the sequence.

number *(n)*	1	2	3	4	5	....	n
term t_n	–3	–6	–9	–12	–15	....	

common difference =

$t_n =$