

17. [Factorization]

Skill 17.1 Factoring by finding the GCF of the coefficients.

MMMaive 11 22 33 44
MMLime 11 22 33 44

- Find the greatest common factor (GCF) of the coefficients in each term. (see skill 5.1, page 49)
- Write the GCF in front of the brackets.
- Write the remaining factors inside the brackets.
- Keep the signs.

Q. Factor $15a - 24$

A. $15a - 24$

$3 \cdot 5a = 15a$ and $3 \cdot 8 = 24$

GCF of 15 and 24 is 3

Remaining factors are 5a and 8.

Write the GCF before the () = $3(5a - 8)$ Keep the sign

a) Factor $4k - 16$ =

b) Factor $4x + 8$ =

$4 \cdot k = 4k$ and $4 \cdot 4 = 16 \Rightarrow$ GCF is 4

c) Factor $6s + 18$ =

d) Factor $3u - 15$ =

e) Factor $9m - 24$ =

f) Factor $14n + 21$ =

g) Factor $2y + 10z$ =

h) Factor $4a - 12b$ =

i) Factor $6d + 14e$ =

j) Factor $16uv - 40u$ =

k) Factor $3m - 6n + 9$ =

l) Factor $4g + 4h - 6$ =

m) Factor $5h^2 - 10i + 25j$ =

n) Factor $6r^2 - 27s + 9t$ =

Skill 17.2 Factoring by finding the GCF of coefficients and variables.

- Find the greatest common factor (GCF) of the coefficients in each term. (see skill 5.1, page 49)
- Find any common factors (CF) from the pronumerals in each term.
- Write the GCF and any other CF in front of the brackets.
- Write the remaining factors inside the brackets.
- Keep the signs.
- Check the result by expanding the brackets.

Q. Factor $18kl - 24k$

A. $18kl - 24k$

$6 \cdot 3 = 18$ and $6 \cdot 4 = 24$

GCF of 18 and 24 is 6

k is common to both terms

Remaining factors are $3l$ and 4 .

Write all CF's before the ()

$= 6k(3l - 4)$

Keep the sign

a) Factor $ab + 5b$

b is common to both terms $= b(a + 5)$

$ab = ba$

b) Factor $de + d$

$=$

c) Factor $7e + ef$

$=$

d) Factor $3st + 4s$

$=$

e) Factor $8ab - 4b$

$=$

f) Factor $15g + 20gh$

$=$

g) Factor $wx - xy$

$=$

h) Factor $2jk + 2kl$

$=$

i) Factor $uv - 3vw$

$=$

j) Factor $8ab + 4bc$

$=$

k) Factor $12qr + 8rs$

$=$

l) Factor $15de - 6ef$

$=$

m) Factor $6xy + 9yz$

$=$

n) Factor $10gh - 25gi$

$=$

Skill 17.3 Factoring to simplify expressions involving large numbers.

- Find the factor repeating in both products.
- Write this number in front of the brackets.
Hint: When both terms are negative the negative sign is taken out as a common factor.
- Write the remaining factors inside the brackets.
- Keep the signs.

Q. Factor and evaluate
 $45 \times 7 + 45 \times 3$

A. $45 \times 7 + 45 \times 3$ *45 is repeating*
 $= 45 \times (7 + 3)$
 $= 45 \times 10$
 $= 450$

a) Factor and evaluate
 $99 \times 99 - 98 \times 99$

$= 99 \times (99 - 98)$
 $= 99 \times 1 = \boxed{99}$

b) Factor and evaluate
 $15 \times 14 + 15 \times 6$

$= 15 \times (14 + 6)$
 $= \dots = \boxed{}$

c) Factor and evaluate
 $987 \times 2 + 987 \times 8$

$= \dots$
 $= \dots = \boxed{}$

d) Factor and evaluate
 $40 \times 8 + 40 \times 12$

$= \dots$
 $= \dots = \boxed{}$

e) Factor and evaluate
 $23 \times 37 + 23 \times 63$

$= \dots$
 $= \dots = \boxed{}$

f) Factor and evaluate
 $25 \times 26 + 25 \times 24$

$= \dots$
 $= \dots = \boxed{}$

g) Factor and evaluate
 $999 \times 9 - 999 \times 8$

$= \dots$
 $= \dots = \boxed{}$

h) Factor and evaluate
 $87 \times 19 - 87 \times 9$

$= \dots$
 $= \dots = \boxed{}$

i) Factor and evaluate
 $-4 \times 14 - 4 \times 6$

$= -4 \times (14 + 6)$ *Both terms are negative so CF is negative*
 $= \dots = \boxed{}$

j) Factor and evaluate
 $-9 \times 33 - 9 \times 67$

$= \dots$
 $= \dots = \boxed{}$

Skill 17.4 Factoring involving squared terms.

- Find the greatest common factor (GCF) of the coefficients in each term. (see skill 5.1, page 49)
- Find any common factors (CF) from the variable in each term.
- Write the GCF and any other CF in front of the brackets.
- Write the remaining factors inside the brackets.
- Check the signs.

Q. Factor $2wx - 12w^2x$

A. $2wx - 12w^2x$

$2 \cdot 1 = 2$ and $2 \cdot 6 = 12$

GCF of 2 and 12 is 2

wx is common to both terms

Remaining factors are 1 and $6w$.

Write all CF's before the ()

$= 2wx(1 - 6w)$

Keep the sign

a) Factor $2j^2k + 5j$

$CF = j$ $= j(2jk + 5)$

Write all CF's before the ()

b) Factor $e^2 + 7e$

$=$

c) Factor $h + 4h^2$

$=$

d) Factor $m^2 - 9m$

$=$

e) Factor $3c - 12c^2$

$=$

f) Factor $4f^2 + 6f$

$=$

g) Factor $fg^2 + f$

$=$

h) Factor $10b - 16ab^2$

$=$

i) Factor $14bc + 2b^2c$

$=$

j) Factor $5r^2s - r^2t$

$=$

k) Factor $vw + 7v^2 - 3vwx$

$=$

l) Factor $8j^2 - 24jk + 12jl$

$=$

m) Factor $f^3g^2 + fg^2$

$=$

n) Factor $p^3q^2 + p^2q + pq$

$=$

Skill 17.5 Factoring negative terms.

MMMaive 11 22 33 44
MMLime 11 22 33 44

- Find the greatest common factor (GCF) of the coefficients in each term. (see skill 5.1, page 49)
- Find any common factors (CF) from the variables in each term.
- Write the GCF and any other CF in front of the brackets.
Hint: When both terms are negative the negative sign is taken out as a common factor.
- Write the remaining factors inside the brackets.
- Check the signs.

Q. Factor $-10r^2 - 5r$

A. $-10r^2 - 5r$

$5 \cdot 2 = 10$ and $5 \cdot 1 = 5$

GCF of 10 and 5 is 5

r is common to both terms

$-$ is common to both terms

Remaining factors are r and 1 .

$= -5r(r + 1)$

Write all CF before the ()

"-" is common to both terms

a) Factor $-7a - 21$

$= -7(a + 3)$

$7 \cdot 1 = 7$ and $7 \cdot 3 = 21 \Rightarrow$ GCF is 7

b) Factor $-4k - 12$

$=$

$4 \cdot 1 = 4$ and $4 \cdot 3 = 12 \Rightarrow$ GCF is

c) Factor $-6g - 15$

$=$

d) Factor $-6e - 14$

$=$

e) Factor $-2h^2 - 6h$

$=$

f) Factor $-8z^2 - 28z$

$=$

g) Factor $-12i^3 - 9ij$

$=$

h) Factor $-t^3 - 5t^2u$

$=$

i) Factor $-2x^3 - 4xy$

$=$

j) Factor $-4m^3 - 12mn^2 + 18m$

$=$

Skill 17.6 Factoring by finding binomial factors.

- Find any common factors (CF).

Hint: It might help to think of common factors that are expressions like $(d + 2)$ as a blob ■.

- Write the CF in front of the brackets.
- Write the remaining factors inside the brackets.
- Keep the signs.
- Check the result by expanding the brackets.

Q. Factor $2(r - 1) - r(r - 1)$

A. $2(r - 1) - r(r - 1)$ Consider $(r - 1) = \blacksquare$
 $= 2 \blacksquare - r \blacksquare$ Keep the sign
 $= \blacksquare (2 - r)$
 $= (r - 1)(2 - r)$

a) Factor $d(d + 2) + 8(d + 2)$ Consider $(d + 2) = \blacksquare$
 $= d \blacksquare + 8 \blacksquare$ Keep the sign
 $= \blacksquare (d + 8)$ = $(d + 2)(d + 8)$

b) Factor $2(h - 3) + h(h - 3)$
 $= 2 \blacksquare + h \blacksquare$
 $=$ =

c) Factor $5(x + 4) + x(x + 4)$
 $=$
 $=$ =

d) Factor $b(b - 7) + 6(b - 7)$
 $=$
 $=$ =

e) Factor $a(a + 2) - 9(a + 2)$
 $=$
 $=$ =

f) Factor $z(z - 5) - (z - 5)$
 $=$
 $=$ =

g) Factor $j^2 + 4j + j + 4$
 $=$
 $=$ =

h) Factor $mn - 2m + 4n - 8$
 $=$
 $=$ =

i) Factor $qs - 3q + st - 3t$
 $=$
 $=$ =

j) Factor $12vw - 6v + 8w - 4$
 $=$
 $=$ =

Skill 17.7 Factoring using the difference of perfect squares.

- Find any common factors (CF) of the terms.
- Write any CF in front of the brackets.
- Use the difference of perfect squares formula.
- Check the result by expanding the brackets.

Expand the brackets

$$\begin{aligned}
 (a + b)(a - b) &= a \times a + a \times -b + b \times a + b \times -b \\
 &= a^2 - ab + ba - b^2 \\
 &= a^2 - b^2
 \end{aligned}$$

Diagram showing the expansion of $(a + b)(a - b)$ with arrows labeled 1, 2, 3, and 4 pointing to the terms $a \times a$, $a \times -b$, $b \times a$, and $b \times -b$ respectively. A callout box labeled "Group like terms" points to the $-ab + ba$ terms.

Q. Factor $5w^2 - 20$

A. $5w^2 - 20$
 $= 5(w^2 - 4)$
 $= 5(w^2 - 2^2)$
 $= 5(w + 2)(w - 2)$

Callouts:
 - "Take out the CF of 5" points to the 5 in the first step.
 - " $4 = 2^2$ " points to the 4 in the second step.
 - "Use $a^2 - b^2 = (a + b)(a - b)$ where $a = w$ and $b = 2$ " points to the final factored form.

a) Factor $c^2 - 81$

$b^2 = 81$
 $b = 9$

Use $a^2 - b^2 = (a + b)(a - b)$

$= c^2 - 9^2 = (c + 9)(c - 9)$

b) Factor $y^2 - 4$

$=$

c) Factor $d^2 - e^2$

$=$

d) Factor $36 - h^2$

$=$

e) Factor $4j^2 - 9$

$=$

f) Factor $2c^2 - 50$

$=$

g) Factor $p^2 - 81q^2$

$=$

h) Factor $80 - 5y^2$

$=$

i) Factor $3d^2 - 27$

$=$

j) Factor $100 - 4k^2$

$=$

Skill 17.8 Factoring four terms by grouping 2 and 2.

- Begin factoring by grouping the 4 terms in 2 groups of 2.
- Take out the CF from the first group of 2 and write it in front of the brackets.
- Take out the CF from the second group of 2 and write it in front of the brackets.
- Keep the signs.
- Factor again by finding the common binomial factor. (see skill 17.6, page 190)
Hint: It might help to think of binomial factors that are expressions like $(d + 2)$ as a blob ■.
- Take out the binomial factor or blob and write it in front of the brackets.
- Write the remaining factors inside the brackets.
- Check the result by expanding the brackets.

Q. Factor $m^2 + 3m + 5m + 15$

A. $m^2 + 3m + 5m + 15$ Keep the sign
 $= m(m + 3) + 5(m + 3)$
 $= m \blacksquare + 5 \blacksquare$ Consider $(m + 3) = \blacksquare$
 $= \blacksquare(m + 3)$
 $= (m + 3)(m + 5)$

a) Factor $c^2 + 8c + 3c + 24$
Group 2 and 2
Factor each group
 $= c(c + 8) + 3(c + 8)$
Factor again
Consider $(c + 8) = \blacksquare$
 $= \blacksquare(c + 3)$ $= (c + 8)(c + 3)$

b) Factor $a^2 + 3a + 2a + 6$
 $= a(a + 3) + 2(a + 3)$
 $=$ $=$

c) Factor $s^2 + 6s + 5s + 30$
 $=$
 $=$ $=$

d) Factor $h^2 + 5h + 4h + 20$
 $=$
 $=$ $=$

e) Factor $v^2 + 7v + 3v + 21$
 $=$
 $=$ $=$

f) Factor $4n + n^2 + 16 + 4n$
 $=$
 $=$ $=$

g) Factor $6t + t^2 - 42 - 7t$
 $=$
 $=$ $=$

h) Factor $4b + 4 - b^2 - b$
 $=$
 $=$ $=$

Skill 17.9 Factoring quadratic trinomials.

- Write two sets of brackets. Because x^2 can only be produced from $x \cdot x$, write the factors of the squared variable in the brackets $(x \quad)(x \quad)$.
- Make a list of all pairs of factors, positive and negative, that produce the whole number.
- From this list determine which pair can be added to get the correct number of x terms.
- Write the result in the brackets with their signs.
- Check the result by expanding the brackets.

Q. Factor $x^2 - 9x + 8$

A. $x^2 - 9x + 8$
 $= (x \quad)(x \quad)$
 $8 = 1 \times 8 = -1 \times (-8) = 2 \times 4 = -2 \times (-4)$
 Only $-1x$ and $-8x$ can make $-9x$
 $= x^2 - 1x - 8x + 8$
 $= (x - 1)(x - 8)$

Write x in the brackets

List the pairs of factors of $+8$

Determine the x terms

AND/OR consider

$= x^2 - 1x - 8x + 8$
 $= x(x - 1) - 8(x - 1)$
 $= x \blacksquare - 8 \blacksquare$
 $= \blacksquare(x - 8)$
 $= (x - 1)(x - 8)$

Group 2 and 2

Factor each group

Factor again

Consider $(x - 1) = \blacksquare$

CHECK
 $(x - 1)(x - 8)$
 $= x^2 - 1x - 8x + 8$
 $= x^2 - 9x + 8 \quad \checkmark$

Expand the brackets

a) Find the missing factor
 $x^2 + 7x + 10$

$10 = 2 \cdot 5 = -2 \cdot (-5)$

$= 5x + 2x = 7x$

$= (x + 5)(x + 2)$

b) Find the missing factor
 $d^2 - 4d + 4$

$4 = 2 \cdot 2 = -2 \cdot (-2)$

$= (d - 2)(\quad)$

c) Find the missing factor
 $s^2 + 4s + 3$

$= (s + 3)(\quad)$

d) Find the missing factor
 $g^2 + 8g + 15$

$= (g + 5)(\quad)$

e) Factor
 $m^2 + 2m - 24$

$-24 = -4 \cdot 6 = 4 \cdot (-6)$

$= 6m - 4m = 2m$

Which pair can be added to get $+2m$?

f) Factor
 $j^2 + 11j + 24$

$= (\quad)$

$= (\quad)$

g) Factor
 $c^2 - 6c + 5$

$= (\quad)$

h) Factor
 $p^2 - 6p - 16$

$= (\quad)$