

18. [Equations]

Skill 18.1 Solving one-step equations by using the inverse operations of + and -

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- Consider the operation used to construct the expression involving the variable.
- Perform the inverse operation on both sides of the equation.

Operation	Inverse Operation	Operation	Inverse Operation
+	-	-	+
$x + 3 = 6$ $x + 3 - 3 = 6 - 3$ $x = 3$		$x - 3 = 6$ $x - 3 + 3 = 6 + 3$ $x = 9$	

Q. Solve for x : $x - 7 = 4$

A.

$$x - 7 = 4$$

Operation: -7

$$x - 7 + 7 = 4 + 7$$

Inverse of -7 is +7

$$x = 11$$

Simplify: $-7 + 7 = 0$

Operation: +2

a) Solve for x : $x + 2 = 5$

$$x + 2 - 2 = 5 - 2$$

$$x = 3$$

b) Solve for x : $x + 4 = 9$

$$x + 4 - 4 = 9 - 4$$

$$x =$$

c) Solve for x : $x + 6 = 9$

$$x =$$

d) Solve for x : $x + 4 = 2$

$$x =$$

e) Solve for x : $x + 7 = -3$

$$x =$$

f) Solve for x : $5 + x = 2$

$$x =$$

g) Solve for x : $x - 3 = 5$

$$x - 3 + 3 = 5 + 3$$

$$x = 8$$

h) Solve for x : $x - 8 = 4$

$$x - 8 + 8 = 4 + 8$$

$$x =$$

i) Solve for x : $x - 7 = 9$

$$x =$$

j) Solve for x : $x - 2 = 14$

$$x =$$

k) Solve for x : $x - 7 = 7$

$$x =$$

l) Solve for x : $x - 9 = 12$

$$x =$$

m) Solve for x : $8 - x = 4$

$$x =$$

n) Solve for x : $x - 5 = -7$

$$x =$$

o) Solve for x : $6 - x = 9$

$$x =$$

Skill 18.2 Solving one-step equations by using the inverse operations of \cdot and \div

- Consider the operation used to construct the expression involving the variable.
- Perform the inverse operation on both sides of the equation.

Operation	Inverse Operation	Operation	Inverse Operation
\cdot	\div	\div	\cdot
$3x = 6$		$\frac{x}{3} = 6$	
$\frac{3x}{3} = \frac{6}{3}$		$\frac{x}{3} \cdot 3 = 6 \cdot 3$	
$x = 2$		$x = 18$	

Q. Solve for x : $\frac{x}{3} = 5$

A. $\frac{x}{3} = 5$ Operation: $\div 3$

$\frac{x}{3} \cdot 3 = 5 \cdot 3$ Inverse of $\div 3$ is $\cdot 3$
 $x = 15$

a) Solve for x : $\frac{x}{7} = 4$ Operation: $\div 7$

$\frac{x}{7} \cdot 7 = 4 \cdot 7$

Inverse of $\div 7$ is $\cdot 7$

$x =$ 28

b) Solve for x : $\frac{x}{3} = 3$

$x =$

c) Solve for x : $\frac{x}{2} = 3$

$x =$

d) Solve for x : $4x = 16$ Operation: $\cdot 4$

$\frac{1}{4}x = \frac{16}{4}$

Inverse of $\cdot 4$ is $\div 4$

Simplify: $\div 4$

$x =$

e) Solve for x : $3x = 12$

$x =$

f) Solve for x : $2x = 14$

$x =$

g) Solve for x : $\frac{x}{2} = 6$

$x =$

h) Solve for x : $\frac{x}{5} = 2$

$x =$

i) Solve for x : $\frac{x}{8} = 6$

$x =$

j) Solve for x : $3x = 27$

$x =$

k) Solve for x : $4x = 28$

$x =$

l) Solve for x : $5x = 45$

$x =$

Skill 18.3 Solving two-step equations by using the inverse operations of +, -, · and ÷

- To isolate the variable (x) perform the inverse operations, in order, to both sides of the equation.

Q. Solve for x: $5 + \frac{2x}{3} = 1$

A. $5 + \frac{2x}{3} = 1$

$5 - 5 + \frac{2x}{3} = 1 - 5$ Inverse of + 5 is - 5

$\frac{2x}{\cancel{3}} \cdot \cancel{3} = -4 \cdot 3$ Inverse of ÷ 3 is · 3

$\frac{2x}{2} = \frac{-12}{2}$ Inverse of · 2 is ÷ 2

$x = -6$

a) Solve for x: $4x - 1 = 11$

b) Solve for x: $7 + 3x = 22$

c) Solve for x: $2x + 7 = -3$

Inverse of - 1 is + 1

$4x - 1 + 1 = 11 + 1$

$7 - 7 + 3x = 22 - 7$

$ = $

$4x = 12$

$3x = $

$ = $

Inverse of · 4 is ÷ 4

$\frac{\cancel{4}x}{\cancel{1}} = \frac{12}{\cancel{1}}$

$= $

$= $

$x = \boxed{3}$

$x = \boxed{}$

$x = \boxed{}$

d) Solve for x: $5x - 1 = 24$

e) Solve for x: $15 + 10x = 45$

f) Solve for x: $3x + 12 = 3$

$=$

$=$

$=$

$=$

$=$

$=$

$=$

$=$

$=$

$x = \boxed{}$

$x = \boxed{}$

$x = \boxed{}$

g) Solve for x: $\frac{x}{4} + 3 = 5$

h) Solve for x: $\frac{x}{5} + 3 = 1$

i) Solve for x: $\frac{5x}{2} - 3 = -1$

$\frac{x}{4} + 3 - 3 = 5 - 3$

$=$

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$x = \boxed{}$

$x = \boxed{}$

$x = \boxed{}$

Skill 18.4 Solving equations by first expanding the brackets.

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- Expand the brackets.
- To isolate the variable (x) perform the inverse operations, in order, to both sides of the equation.

Q. Solve for x : $6(2 - x) = -18$

A. $6(2 - x) = -18$ Expand the ()

$$12 - 6x = -18$$

$$12 - 12 - 6x = -18 - 12$$
 Inverse of +12 is -12

$$-6x = -30$$

$$\frac{-6x}{-6} = \frac{-30}{-6}$$
 Inverse of $\cdot -6$ is $\div -6$

$$x = 5$$

a) Solve for x : $3(x - 2) = 12$

Expand the ()

$$3x - 6 = 12$$

Inverse of -6 is $+6$

$$3x - 6 + 6 = 12 + 6$$

Inverse of $\cdot 3$ is $\div 3$

$$\frac{3x}{3} = \frac{12+6}{3}$$

$$x = \boxed{4}$$

b) Solve for x : $3(2 + x) = 21$

$$=$$

$$=$$

$$=$$

$$x = \boxed{}$$

c) Solve for x : $2(x - 3) = 14$

$$=$$

$$=$$

$$=$$

$$x = \boxed{}$$

d) Solve for x : $5(1 + x) = 20$

$$=$$

$$=$$

$$=$$

$$x = \boxed{}$$

e) Solve for x : $7(2 + x) = 35$

$$=$$

$$=$$

$$=$$

$$x = \boxed{}$$

f) Solve for x : $4(x - 3) = 4$

$$=$$

$$=$$

$$=$$

$$x = \boxed{}$$

g) Solve for x : $4(x - 5) = 8$

$$=$$

$$=$$

$$=$$

$$x = \boxed{}$$

h) Solve for x : $2(9 - x) = 8$

$$=$$

$$=$$

$$=$$

$$x = \boxed{}$$

i) Solve for x : $3(2x - 3) = 15$

$$=$$

$$=$$

$$=$$

$$x = \boxed{}$$

Skill 18.5 Solving equations with variables in more than one place.

- If necessary, expand the brackets. (see skill 18.4, page 198)
- Combine all variables on one side of the equation by using inverse operations.
- To isolate the variable (x) perform the inverse operations, in order, to both sides of the equation.

Q. Solve for x : $x = 3x + 12$

A.

$$x = 3x + 12$$

Combine x 's: $-3x$

$$x - 3x = 3x - 3x + 12$$

$$-2x = 12$$

Operation: $\cdot (-2)$

$$\frac{-2x}{-2} = \frac{12}{-2}$$

Inverse of $\cdot (-2)$ is $\div (-2)$

$$x = -6$$

a) Solve for x : $6 - 3x = 3x$

$$6 - 6 - 3x = 3x - 6$$

$$-3x - 3x = 3x - 3x - 6$$

$$\frac{-6x}{-6} = \frac{-6}{-6}$$

Inverse of $\cdot (-6)$ is $\div (-6)$

$$x = \boxed{1}$$

b) Solve for x : $6x + 4 = 8x$

$$=$$

$$=$$

$$=$$

$$x = \boxed{}$$

c) Solve for x : $25 - 4x = x$

$$=$$

$$=$$

$$=$$

$$x = \boxed{}$$

d) Solve for x : $6x - 4 = 5x$

$$=$$

$$=$$

$$=$$

$$x = \boxed{}$$

e) Solve for x : $15 - 3x = 2x$

$$=$$

$$=$$

$$=$$

$$x = \boxed{}$$

f) Solve for x : $7x - 24 = 4x$

$$=$$

$$=$$

$$=$$

$$x = \boxed{}$$

g) Solve for x : $4x + 2(3x - 4) = 22$

Expand the $()$

Combine the variables

$$4x + 6x - 8 = 22$$

$$10x - 8 + 8 = 22 + 8$$

$$\frac{10x}{10} = \frac{30}{10}$$

$$x = \boxed{}$$

h) Solve for x : $2x + 3(4x - 3) = 19$

$$=$$

$$=$$

$$=$$

$$x = \boxed{}$$

i) Solve for x : $2(x - 3) - 3x = -12$

$$=$$

$$=$$

$$=$$

$$x = \boxed{}$$

Skill 18.6 Solving equations involving fractions.

- Use inverse operations rules to isolate any fractions.
- Rewrite all expressions as fractions if necessary.
- Cross multiply. (see skill 9.5, page 102)
- Combine all variables on one side of the equation by using inverse operations. (see skill 18.5, page 199)
- To isolate the variable (x) perform the inverse operations, in order, to both sides of the equation.

Q. Solve for x : $\frac{x}{3} = x + 4$

A.

$$\frac{x}{3} = x + 4$$

$$\frac{x}{3} \times \frac{x+4}{1} \quad \text{Cross multiply}$$

$$x = 3(x + 4)$$

$$x = 3x + 12$$

$$x - 3x = 3x - 3x + 12 \quad \text{Combine } x^{\text{'s}}: -3x$$

$$-2x = 12 \quad \text{Operation: } \cdot (-2)$$

$$\frac{-2x}{-2} = \frac{12}{-2}$$

$$x = -6$$

a) Solve for x : $\frac{x}{4} - 10 = -x$

$$\frac{x}{4} - 10 + 10 = -x + 10 \quad \text{Isolate the fraction}$$

$$\frac{x}{4} \times \frac{-x+10}{1} \quad \text{Rewrite expression as fraction}$$

$$x = 4(-x + 10)$$

$$x + 4x = -4x + 4x + 40$$

$$5x = 40$$

$$x = \boxed{}$$

b) Solve for x : $\frac{18}{x} = 2$

$$=$$

$$=$$

$$=$$

$$x = \boxed{}$$

c) Solve for x : $\frac{20 - 2x}{3} = 2$

$$=$$

$$=$$

$$=$$

$$=$$

$$x = \boxed{}$$

d) Solve for x : $\frac{2x}{5} = x - 3$

$$=$$

$$=$$

$$=$$

$$=$$

$$x = \boxed{}$$

e) Solve for x : $\frac{x+4}{3} = \frac{10-x}{4}$

$$=$$

$$=$$

$$=$$

$$=$$

$$x = \boxed{}$$

f) Solve for x : $\frac{2x}{3} + 10 = 4x$

$$=$$

$$=$$

$$=$$

$$=$$

$$x = \boxed{}$$

Skill 18.7 Solving inequalities (1).

- Manipulate the inequality in the same way as you would an equation.

EXCEPT:

- When both sides are multiplied or divided by a negative number, reverse the inequality signs. $<$ becomes $>$ and \leq becomes \geq .

Q. Solve the inequality:
 $4x - 7 \leq 5$

A. $4x - 7 \leq 5$
 $4x \xrightarrow{-7} \xrightarrow{+7} \leq 5 + 7$
 $\frac{4x}{4} \leq \frac{12}{4}$
 $x \leq 3$

a) Solve the inequality:
 $2x - 5 \geq 15$

\geq

 \geq

 $x \geq$

b) Solve the inequality:
 $3x - 8 < 7$

$<$

 $<$

 $x <$

c) Solve the inequality:
 $2x + 6 \leq 10$

\leq

 \leq

 []

d) Solve the inequality:
 $7(-x + 3) < 7$

$-7x + 21 < 7$

 $-7x + \cancel{21} - \cancel{21} < 7 - 21$

 $-7x < -14$

 $\frac{-7x}{-7} < \frac{-14}{-7}$
 $x > 2$

Division negative so reverse $<$ sign

e) Solve the inequality:
 $3(2x - 4) + 19 > 25$

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 $>$

 $>$

 $>$

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f) Solve the inequality:
 $20 \geq 5(7 - 2x) - 35$

\geq

 \geq

 \geq

 \geq

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g) Solve the inequality:
 $-5(x + 7) \geq 10$

\geq

 \geq

 \geq

 \geq

 []

h) Solve the inequality:
 $12 \geq 2(x + 3) - 10$

\geq

 \geq

 \geq

 \geq

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i) Solve the inequality:
 $4 < 2(3 - 2x) - 10$

$<$

 $<$

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Skill 18.7 Solving inequalities (2).

MMMaube 1 1 2 2 3 3 4 4
MMLime 1 1 2 2 3 3 4 4

j) Solve the inequality:

$$\frac{x}{2} - 4 \leq 8$$

 \leq

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k) Solve the inequality:

$$\frac{x}{4} + 7 < 11$$

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l) Solve the inequality:

$$-8 + \frac{x}{3} > -1$$

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m) Solve the inequality:

$$\frac{3(x+4)}{2} > 15$$

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n) Solve the inequality:

$$\frac{4(x+1)}{4} \geq 10$$

 \geq

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o) Solve the inequality:

$$\frac{2(6-x)}{3} \geq 8$$

 \geq

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p) Solve the inequality:

$$2x + \frac{x}{3} \leq 21$$

 \leq

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q) Solve the inequality:

$$\frac{2x}{3} - x > -1$$

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r) Solve the inequality:

$$-5x - \frac{x}{2} < -33$$

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Skill 18.8 Solving quadratic equations.

- Make either factor equal zero. Use the zero multiplication property.

$$a \cdot 0 = 0 \text{ and } 0 \cdot a = 0$$

Hint: A quadratic equation always has 2 solutions.

Q. Solve for x :
 $(x - 8)(x + 9) = 0$

If either $(x - 8) = 0$
or $(x + 9) = 0$
then
 $(x - 8)(x + 9) = 0$

A. $(x - 8)(x + 9) = 0$
 $x - 8 + 8 = 0 + 8$
 $x = 8$
OR
 $x + 9 - 9 = 0 - 9$
 $x = -9$
8, -9

Check:
If $x = 8$
 $(8 - 8)(8 + 9) = 0$
 $0 \cdot 17 = 0$ is true
If $x = -9$
 $(-9 - 8)(-9 + 9) = 0$
 $-17 \cdot 0 = 0$ is true

a) Solve for x :
 $(x - 6)(x - 5) = 0$

Make $(x - 6) = 0$

Make $(x - 5) = 0$

If $x - 6 = 0$, then

If $x - 5 = 0$, then

$x - 6 + 6 = 0 + 6$

$x - 5 + 5 = 0 + 5$

$x = 6$

$x = 5$

6, 5

b) Solve for x :
 $(x + 7)(x - 2) = 0$

If $x + 7 = 0$, then

If $x - 2 = 0$, then

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c) Solve for x :
 $(x - 4)(x + 7) = 0$

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d) Solve for x :
 $(x + 3)(x + 4) = 0$

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e) Solve for x : $x(x - 8) = 0$

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f) Solve for x : $x(x + 3) = 0$

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g) Solve for x : $x^2 = 81$

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h) Solve for x : $x^2 = 121$

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EITHER

- Find the value of one of the variables in relation to the other.
- Substitute this value of the variable into the other equation.
- Solve for one variable.
- Substitute the result into either equation to find the second variable.

OR

- Add or subtract the equations together to eliminate one of the variables.

Q. Solve the system of equations:

$$\begin{aligned} 2x + 3y &= 3 \\ x + 3y &= 6 \end{aligned}$$

A. $2x + 3y = 3$ (1)

$x + 3y = 6$ (2)

Eliminate 'y' by subtracting (1) - (2)

$$x = -3$$

$$-3 + 3y = 6$$

Substitute $x = -3$ into (2)

$$-3 + 3 + 3y = 6 + 3$$

$$3y = 9$$

$$\frac{3y}{3} = \frac{9}{3}$$

$$y = 3$$

(-3,3)

a) Solve the system of equations:

$$\begin{aligned} y &= 3x - 9 \\ x &= 4 \end{aligned}$$

Substitute $x = 4$ into (1)

$$y = 3 \cdot 4 - 9$$

$$y = 12 - 9$$

$$y = 3$$

(4,3)

b) Solve the system of equations:

$$\begin{aligned} x + y &= 5 \\ y &= x + 1 \end{aligned}$$

Substitute $y = x + 1$ into (1)

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c) Solve the system of equations:

$$\begin{aligned} y &= 2x + 1 \\ y &= 3x - 2 \end{aligned}$$

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d) Solve the system of equations:

$$\begin{aligned} 4 &= 2x + y \\ x - 5 &= y \end{aligned}$$

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e) Solve the system of equations:

$$\begin{aligned} x + y &= 1 \\ x - y &= 3 \end{aligned}$$

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f) Solve the system of equations:

$$\begin{aligned} x - y &= 2 \\ 3x + y &= 14 \end{aligned}$$

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Skill 18.10 Solving quadratic equations by factoring.

- Factor the expression. (see skill 17.4, page 188 and skill 17.7, page 191)
- Make either factor equal zero. Use the zero multiplication property.
 $a \cdot 0 = 0$ and $0 \cdot a = 0$

Q. Solve for x : $x^2 + 7x = 0$

A. $x^2 + 7x = 0$

Factor

$$x(x + 7) = 0$$

$$x = 0$$

OR

$$x + 7 - 7 = 0 - 7$$

$$x = -7$$

$$0, -7$$

If either
 $x = 0$
or $(x + 7) = 0$
then $x^2 + 7x = 0$

a) Solve for x : $x^2 - 16 = 0$

Factor

$$(x + 4)(x - 4) = 0$$

$$\text{so } x = 4 \text{ or } x = -4$$

If either
 $(x + 4) = 0$
or $(x - 4) = 0$
then $x^2 - 16 = 0$

b) Solve for x : $x^2 - 4 = 0$

c) Solve for x : $x^2 - 2x = 0$

d) Solve for x : $x^2 - 3x = 0$

e) Solve for x : $x^2 + 4x = 0$

f) Solve for x : $x^2 + 5x = 0$

g) Solve for x : $x^2 - 64 = 0$

h) Solve for x : $x^2 - 144 = 0$