

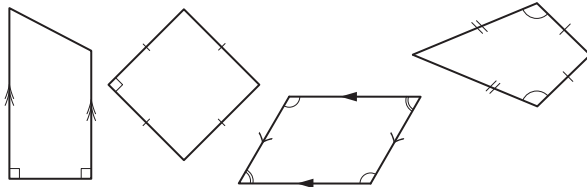
# 20. [Shapes]

## Skill 20.1 Recognizing polygons, quadrilaterals and triangles.

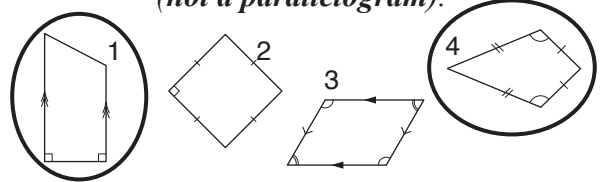
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- Consider the definition of a polygon. (see Glossary, page 422)
- Consider the properties of a parallelogram:
  - both pairs of opposite sides are parallel and equal in length.
- Consider the properties of an isosceles triangle:
  - two sides and two corresponding angles are equal.

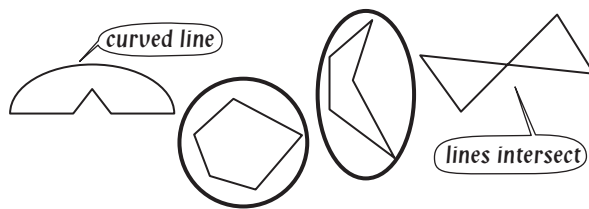
**Q.** Circle the shapes that are **not** parallelograms.



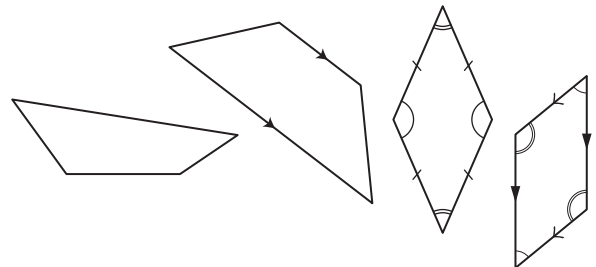
- A.** 1<sup>st</sup> shape - has only two opposite sides parallel (**not a parallelogram**).  
 2<sup>nd</sup> shape - has both pairs of opposite sides equal in length (parallelogram).  
 3<sup>rd</sup> shape - has both pairs of opposite sides parallel (parallelogram).  
 4<sup>th</sup> shape - doesn't have any parallel sides (**not a parallelogram**).



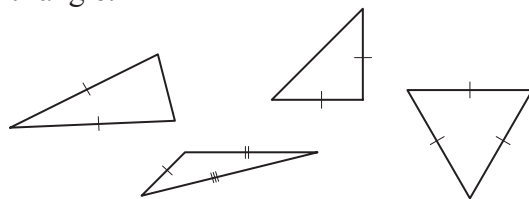
**a)** Circle the shapes that are polygons.



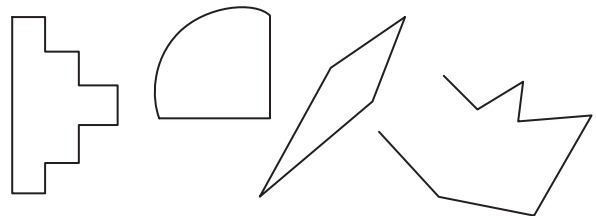
**b)** Circle the shapes that are **not** parallelograms.



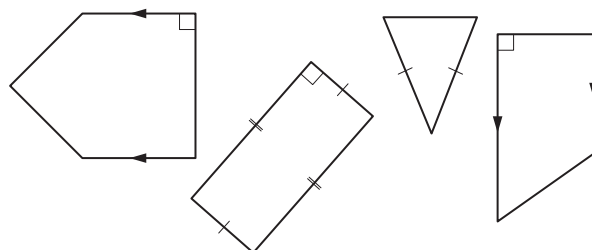
**c)** Circle the shape that is **not** an isosceles triangle.



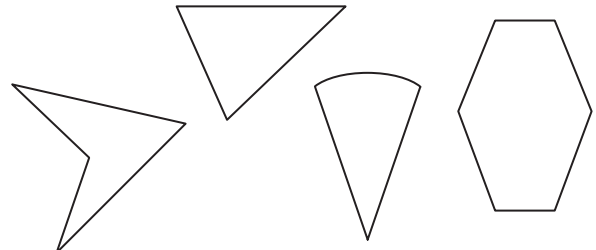
**d)** Circle the shapes that are **not** polygons.



**e)** Circle the shape that is a parallelogram.



**f)** Circle the shape that is **not** a polygon.



## Skill 20.2 Classifying triangles.

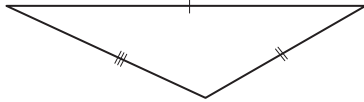
- Look for equal sides or equal angles.
- Look at the types of angles inside the triangle.

Sides and angles	Triangle type
no equal sides/angles	<b>scalene</b>
two equal sides/angles	<b>isosceles</b>
three equal sides/angles	<b>equilateral</b>

Angles	Triangle type
all acute angles	<b>acute</b>
one right angle	<b>right</b>
one obtuse angle	<b>obtuse</b>

**Q.** Which two options describe this triangle?

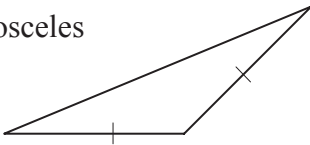
- A) scalene  
B) equilateral  
C) obtuse



- A.** A) scalene (no equal sides/angles)  $\Rightarrow$  true  
B) equilateral (all equal sides)  $\Rightarrow$  false  
C) obtuse (1 obtuse angle)  $\Rightarrow$  true  
The answer is **A** and **C**.

**a)** Which two options describe this triangle?

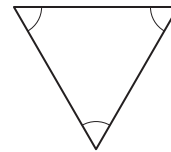
- A) right  
B) obtuse  
C) isosceles



**B** and **C**

**b)** Which two options describe this triangle?

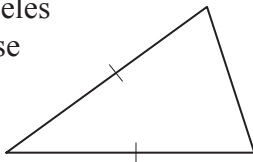
- A) equilateral  
B) scalene  
C) acute



and

**c)** Which two options describe this triangle?

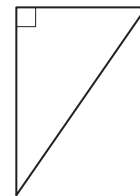
- A) acute  
B) isosceles  
C) obtuse



and

**d)** Which two options describe this triangle?

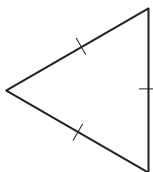
- A) acute  
B) right  
C) scalene



and

**e)** Which two options describe this triangle?

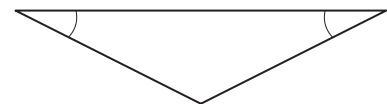
- A) acute  
B) scalene  
C) equilateral



and

**f)** Which two options describe this triangle?

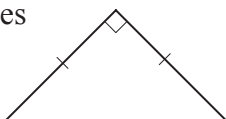
- A) isosceles  
B) obtuse  
C) right



and

**g)** Which two options describe this triangle?

- A) acute  
B) right  
C) isosceles



and

**h)** Which two options describe this triangle?

- A) scalene  
B) isosceles  
C) acute



and

**Skill 20.3** Describing the properties of quadrilaterals.

- Consider the properties of squares, rectangles, rhombi, parallelograms, kites and trapezoids. (see Glossary)

**Q.** I am a quadrilateral whose diagonals are not equal in length and bisect each other at right angles. What am I?

- A) square
- B) parallelogram
- C) rhombus
- D) kite

- A.** A) diagonals are equal  $\Rightarrow$  A false  
 B) diagonals do not bisect each other at right angles  $\Rightarrow$  B false  
 C) diagonals are not equal and bisect each other at right angles  $\Rightarrow$  C true  
 D) diagonals don't bisect each other  $\Rightarrow$  D false

The answer is **C**.

**a)** I am a two-dimensional shape with 4 sides. My diagonals are not equal in length and bisect each other but not at right angles. What am I?

- A) rhombus
- B) parallelogram
- C) kite
- D) trapezoid

**B**

**b)** I am a quadrilateral with both pairs of opposite sides parallel and diagonals equal in length. What am I?

- A) rhombus
- B) trapezoid
- C) parallelogram
- D) rectangle

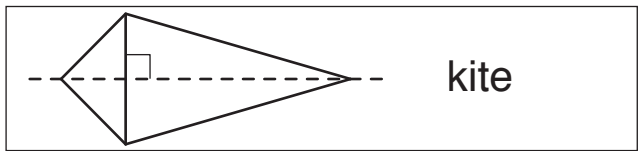
**c)** I am a two-dimensional shape with 4 sides. Adjacent angles are not equal and I have two axes of symmetry. What am I?

- A) trapezoid
- B) kite
- C) rhombus
- D) rectangle

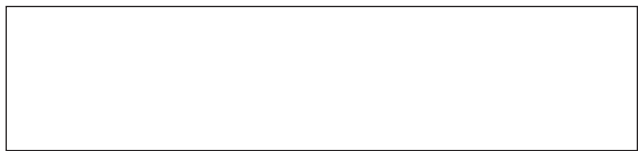
**d)** I am a quadrilateral with both pairs of opposite sides equal in length, but no axis of symmetry. What am I?

- A) square
- B) trapezoid
- C) parallelogram
- D) rhombus

**e)** Draw and name the quadrilateral which has diagonals that are perpendicular, but has only one axis of symmetry.



**f)** Draw and name the quadrilateral which has diagonals that are equal in length and bisect each other at right angles.



**g)** Draw and name the quadrilateral which has pairs of opposite angles that are equal, but does not have any axis of symmetry.



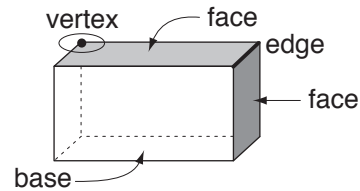
**h)** Draw and name the quadrilateral which has diagonals that bisect each other at right angles and has two axes of symmetry.



## Skill 20.4 Describing the properties of three-dimensional shapes.

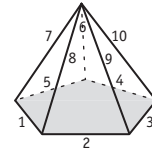
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- Count the number of:
  - faces
  - edges
  - vertices (points/corners)

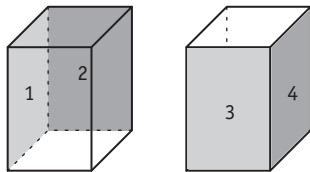


**Q.** How many edges does a pentagonal pyramid have?

**A.** Count the number of edges, or straight lines in the pyramid: 5 edges in the base and 5 vertical edges.  
The answer is **10**



**a)** Of the 6 faces of a square prism, how many are rectangles?

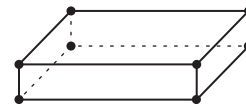


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**b)** Of the 7 faces of a pentagonal prism, how many are rectangles?

**c)** How many vertices are there on a triangular prism?

**d)** How many vertices are there on a rectangular prism?



**e)** How many edges are there on a tetrahedron?

**f)** How many edges are there on a rectangular pyramid?

**g)** How many faces are there on a pentagonal pyramid?

**h)** How many faces are there on a triangular prism?

**i)** Sketch and name the three-dimensional shape that has 6 faces, all of which are squares.

**j)** Sketch and name the three-dimensional solid that has 4 faces, all of which are triangles.

**k)** Sketch and name the three-dimensional shape that has 6 faces, five of which are triangles.

**l)** Sketch and name the three-dimensional solid that has 6 faces, all of which are rectangles.

Euler's formula for any polyhedra:

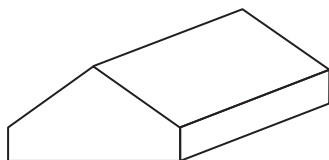
**Edges = Vertices + Faces - 2**

OR

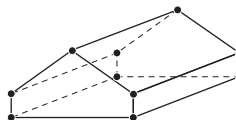
**E = V + F - 2**

**Q.** Euler's formula,  $E = V + F - 2$  defines the relationship between Edges, Vertices and Faces of any polyhedron. Verify Euler's formula for this solid:

$$\boxed{\phantom{00}} = \boxed{\phantom{00}} + \boxed{\phantom{00}} - 2$$



**A.**



$$E = 15$$

$$V = 10$$

$$F = 7$$

$$E = V + F - 2$$

$$15 = 10 + 7 - 2$$

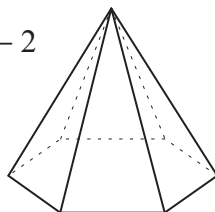
$$15 = 15 \text{ (true)}$$

Substitute 15, 10, 7 into Euler's formula

$$\boxed{15} = \boxed{10} + \boxed{7} - 2$$

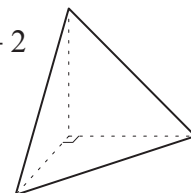
**a)** Euler's formula,  $E = V + F - 2$  defines the relationship between Edges, Vertices and Faces of any polyhedron. Verify Euler's formula for a hexagonal pyramid:

$$\boxed{12} = \boxed{7} + \boxed{7} - 2$$



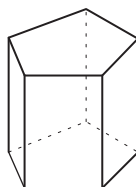
**b)** Euler's formula,  $E = V + F - 2$  defines the relationship between Edges, Vertices and Faces of any polyhedron. Verify Euler's formula for a triangular pyramid:

$$\boxed{\phantom{00}} = \boxed{\phantom{00}} + \boxed{\phantom{00}} - 2$$



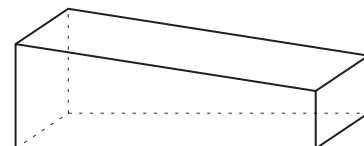
**c)** Euler's formula,  $E = V + F - 2$  defines the relationship between Edges, Vertices and Faces of any polyhedron. Verify Euler's formula for a pentagonal prism:

$$\boxed{\phantom{00}} = \boxed{\phantom{00}} + \boxed{\phantom{00}} - 2$$



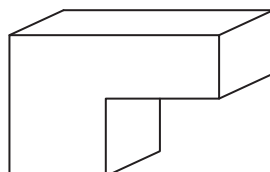
**d)** Euler's formula,  $E = V + F - 2$  defines the relationship between Edges, Vertices and Faces of any polyhedron. Verify Euler's formula for this prism:

$$\boxed{\phantom{00}} = \boxed{\phantom{00}} + \boxed{\phantom{00}} - 2$$



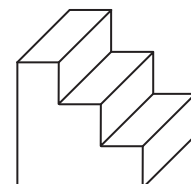
**e)** Euler's formula,  $E = V + F - 2$  defines the relationship between Edges, Vertices and Faces of any polyhedron. Verify Euler's formula for this prism:

$$\boxed{\phantom{00}} = \boxed{\phantom{00}} + \boxed{\phantom{00}} - 2$$



**f)** Euler's formula,  $E = V + F - 2$  defines the relationship between Edges, Vertices and Faces of any polyhedron. Verify Euler's formula for this prism:

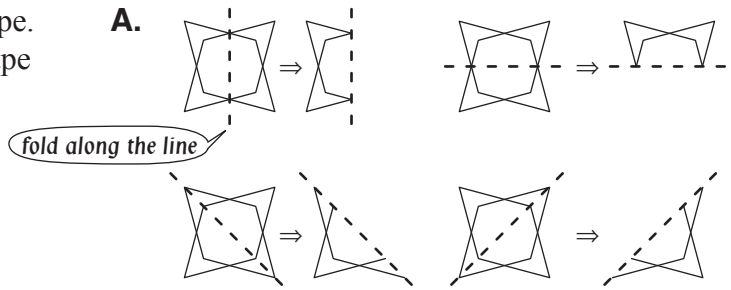
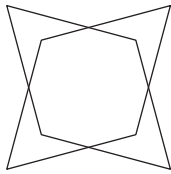
$$\boxed{\phantom{00}} = \boxed{\phantom{00}} + \boxed{\phantom{00}} - 2$$



**Skill 20.6** Recognizing line symmetry in two-dimensional shapes.

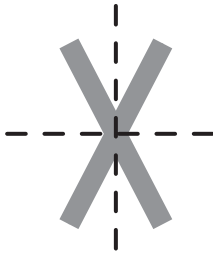
- Imagine along which line the shape can be folded to have one part fit exactly over the other part.

**Q.** Draw all the axes of symmetry for this shape. How many axes of symmetry does this shape have?



**4 axes of symmetry**

**a)** Draw all the axes of symmetry for this shape. How many axes of symmetry does this shape have?



**b)** Draw all the axes of symmetry for this shape. How many axes of symmetry does this shape have?



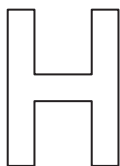
**c)** Draw all the axes of symmetry for this shape. How many axes of symmetry does this shape have?



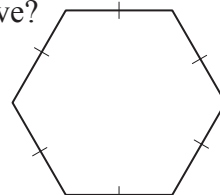
**d)** Draw all the axes of symmetry for this shape. How many axes of symmetry does this shape have?



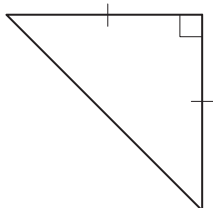
**e)** Draw all the axes of symmetry for this shape. How many axes of symmetry does this shape have?



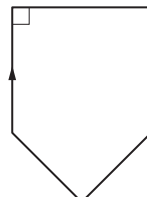
**f)** Draw all the axes of symmetry for this shape. How many axes of symmetry does this regular hexagon have?



**g)** Draw and name the shape formed if this triangle is folded along its axis of symmetry.

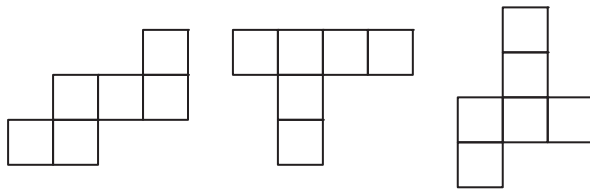


**h)** Draw and name the shape formed if this shape is folded along its axis of symmetry.



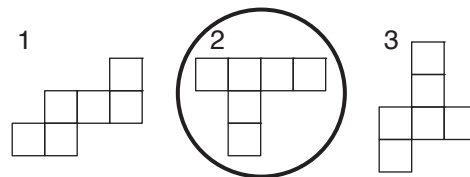
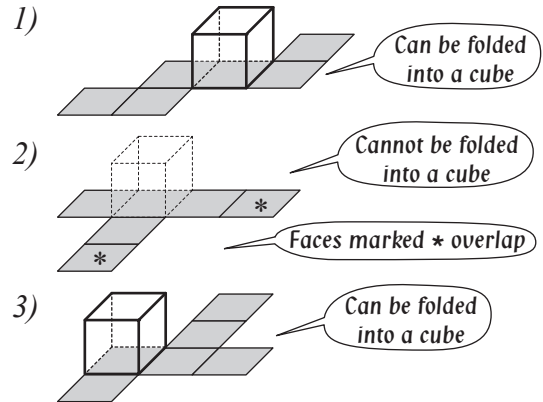
- Identify the shapes in the net. | OR
- Imagine the shape folded. | • Make a model by tracing, cutting out and folding the net.

**Q.** Circle the net below that **cannot** be folded to make a model of a cube.

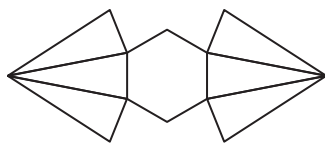


**A.** Enlarge, trace and cut out the shape, folding to try to form a cube.

OR Imagine the shape folded:

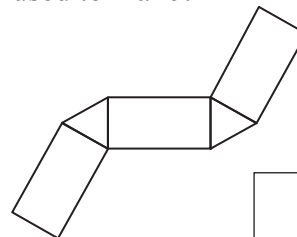


**a)** What three-dimensional shape can this net be used to make?

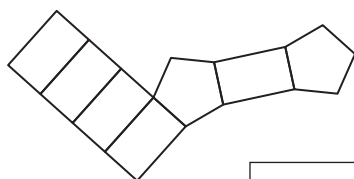


hexagonal pyramid

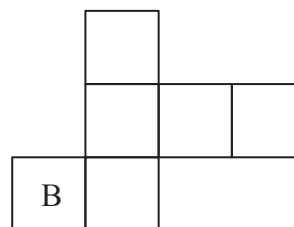
**b)** What three-dimensional shape can this net be used to make?



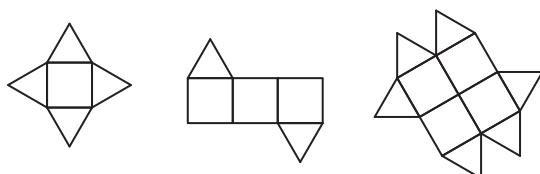
**c)** What three-dimensional shape can this net be used to make?



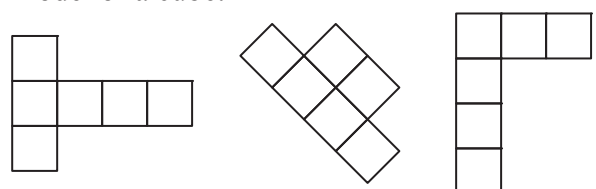
**d)** On this net of a cube, a face is marked B. Label the opposite face with a T.



**e)** Circle the net below that **cannot** be folded to make a model of a three-dimensional shape.



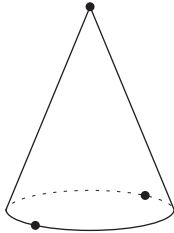
**f)** Circle the net below that **can** be folded to make a model of a cube.



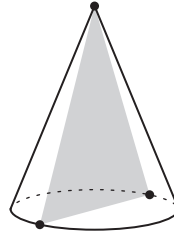
**Skill 20.8** Recognizing the shape of cross sections through three-dimensional shapes.

- To name the shape of a cross section through a 3D shape, imagine that you cut the solid through that section, then separate the two parts and look at the shape of the slice.
  - To draw a cross section of a 3D shape through given points, join these points inside the solid.
- Hint: The cross section of a sphere is always a disc, no matter the angle of the cut.*

**Q.** Draw a cross section through the marked points in this cone. Name the shape drawn.

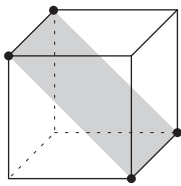


**A.** Join the three points.



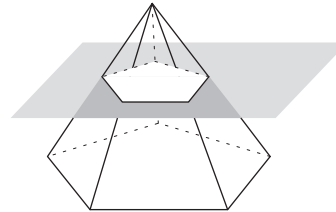
*The shape of the cross section is an isosceles triangle.*

**a)** Name the shape of the cross section drawn through this cube.

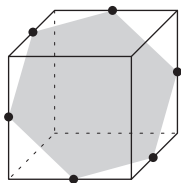


rectangle

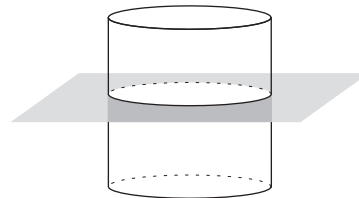
**b)** Name the shape of the cross section drawn through this pyramid.



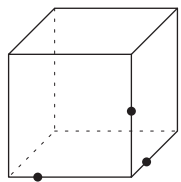

**c)** Name the shape of the cross section drawn through this cube.



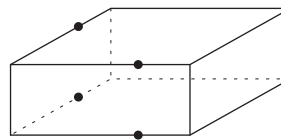

**d)** Name the shape of the cross section drawn through this cylinder.



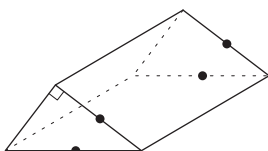

**e)** Draw a cross section through the marked points in this cube. Name the shape drawn.



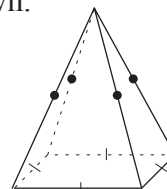

**f)** Draw a cross section through the marked points in this prism. Name the shape drawn.




**g)** Draw a cross section through the marked points in this triangular prism. Name the shape drawn.




**h)** Draw a cross section through the marked points in this square pyramid. Name the shape drawn.

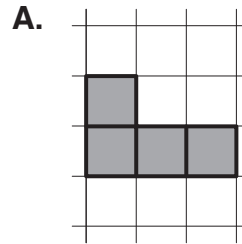
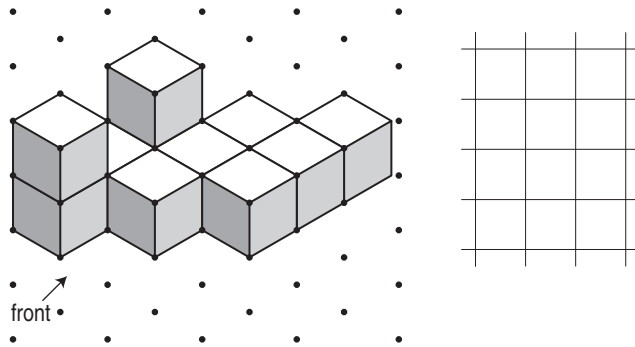




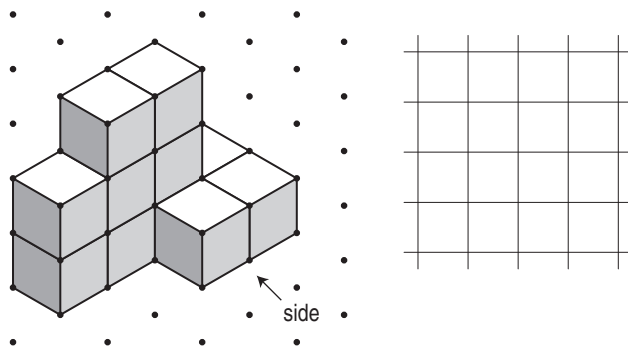
## Skill 20.9 Drawing two-dimensional views of three-dimensional shapes.

- To draw the top view of a 3D shape, imagine what you would see if you were looking at the solid from directly above.
- To draw the side view of a 3D shape, imagine what you would see if you were looking at one of the sides of the solid.
- To draw the front view of a 3D shape, imagine what you would see if you were looking at the front of the solid.

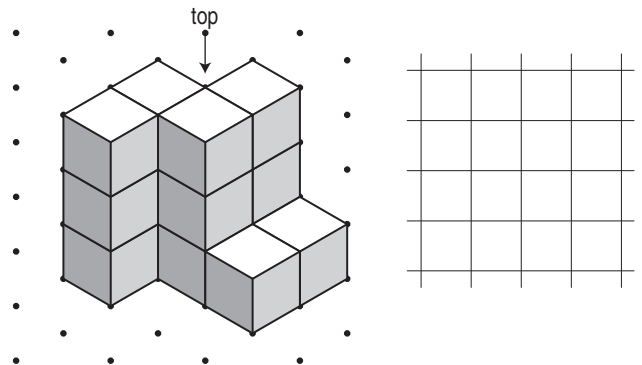
**Q.** Draw the front view of this solid.



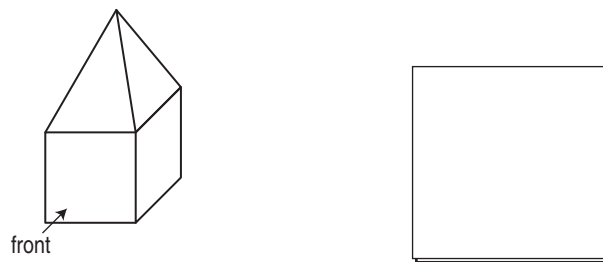
**a)** Draw the side view of this solid.



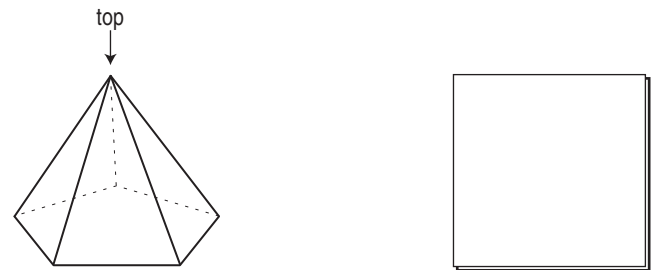
**b)** Draw the top view of this solid.



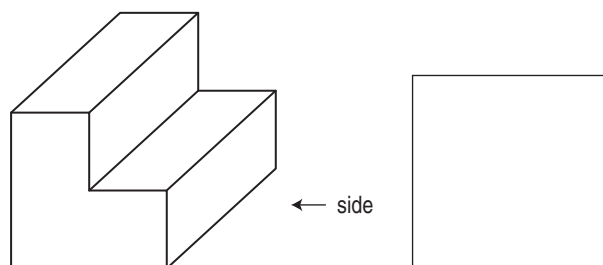
**c)** Sketch the front view of this solid.



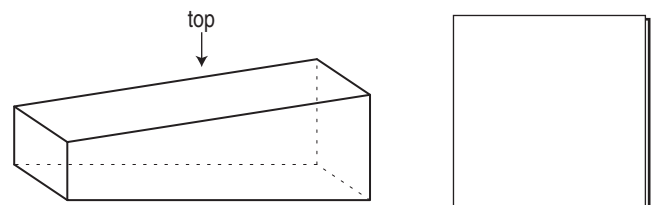
**d)** Sketch the top view of this solid.



**e)** Sketch the side view of this solid.



**f)** Sketch the top view of this solid.



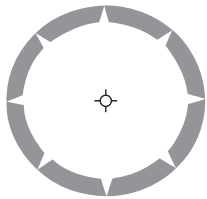
**Skill 20.10** Recognizing rotational symmetry in two-dimensional shapes.

- Try to visualize the shape during a full turn of  $360^\circ$ .
- Count how many times during the full turn the image of the shape exactly covers the original shape.
- This number is called the order of rotational symmetry.

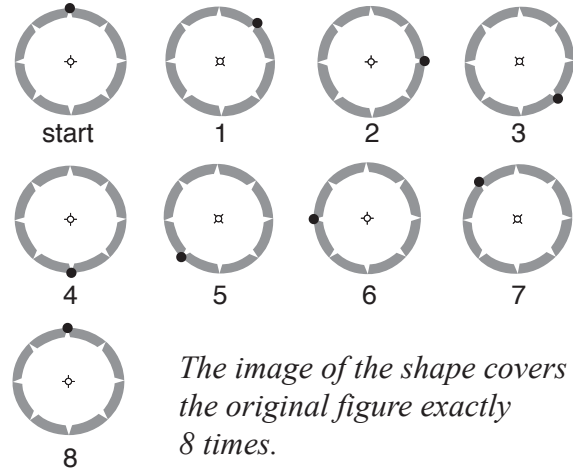
*Hints: A shape does not have rotational symmetry if, during a full rotation of  $360^\circ$ , the image of the shape does not exactly cover the original shape.*

*To count how many times the image of the shape exactly covers the original shape, mark a point on the shape, so you know when the shape has done a complete rotation of  $360^\circ$ .*

**Q.** What is the order of rotational symmetry for this shape?



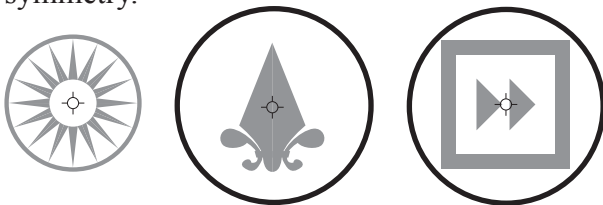
**A.**



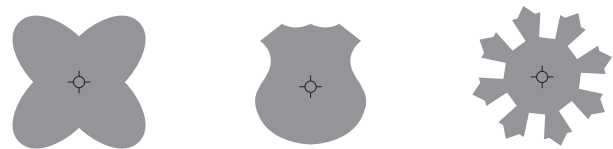
*The image of the shape covers the original figure exactly 8 times.*

*The order of rotational symmetry = 8*

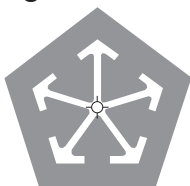
**a)** Circle the shapes that do not have rotational symmetry.



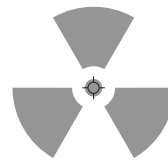
**b)** Circle the shapes that have rotational symmetry.



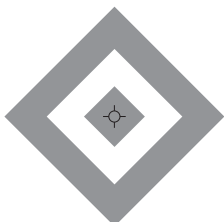
**c)** This shape has rotational symmetry. How many times during a full turn ( $360^\circ$ ) does the image of the shape exactly cover the original figure?



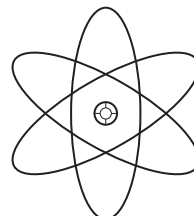

**d)** This shape has rotational symmetry. How many times during a full turn ( $360^\circ$ ) does the image of the shape exactly cover the original figure?




**e)** What is the order of rotational symmetry for this shape?



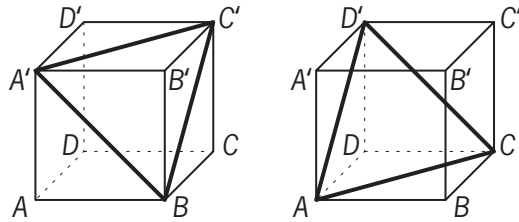

**f)** What is the order of rotational symmetry for this shape?



**Skill 20.11 Finding angle size inside a cube (1).**

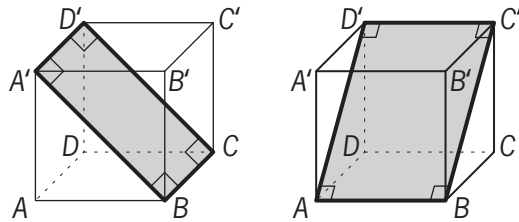
- Consider the definition and the properties of a cube:
  - all faces are squares
  - all edges are equal in length
  - diagonals of all six faces are equal in length.

Example 1:



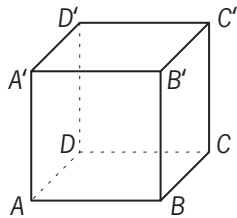
$A'B = BC' = C'A' = AC = CD' = D'A$ , so  $A'BC'$  and  $ACD'$  are equilateral triangles.

Example 2:

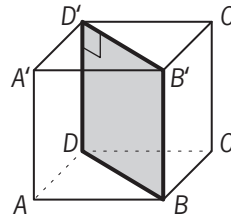


$A'BCD'$  and  $ABC'D'$  are rectangles.

**Q.** How many degrees is  $\angle DD'B'$  in this cube?

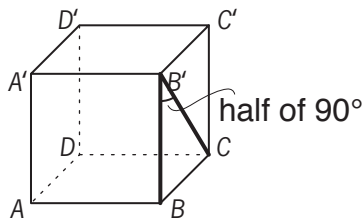


**A.**

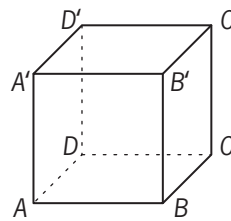


$DBB'D'$  is a rectangle  $\Rightarrow$   
 $\angle DD'B'$  is a right angle  $\Rightarrow$   
 $\angle DD'B' = 90^\circ$

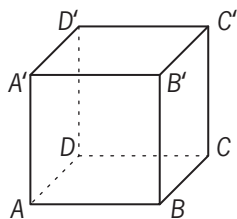
**a)** How many degrees is  $\angle BB'C$  in this cube?



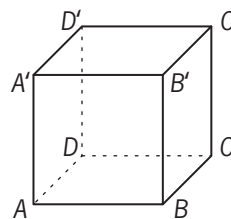

**b)** How many degrees is  $\angle ABC'$  in this cube?



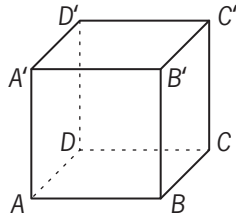

**c)** How many degrees is  $\angle BB'D'$  in this cube?

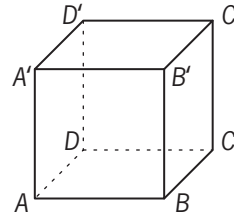


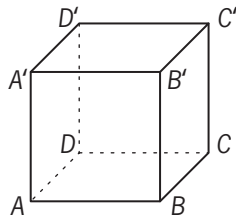

**d)** How many degrees is  $\angle BC'A'$  in this cube?

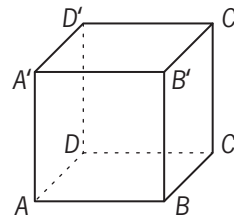


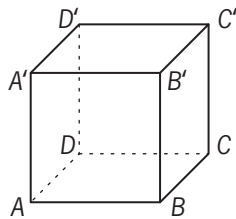
## Skill 20.11 Finding angle size inside a cube (2).

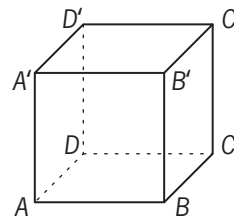
MMMaive 11 22 33 44  
MMLime 11 22 33 44e) How many degrees is  $\angle DD'A'$  in this cube?

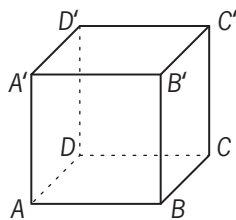
f) How many degrees is  $\angle ADC$  in this cube?

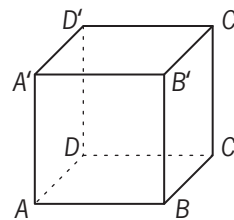
g) How many degrees is  $\angle BDC$  in this cube?

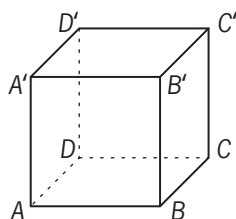
h) How many degrees is  $\angle D'A'B$  in this cube?

i) How many degrees is  $\angle A'BC'$  in this cube?

j) How many degrees is  $\angle C'CD'$  in this cube?

k) How many degrees is  $\angle AD'B'$  in this cube?

l) How many degrees is  $\angle A'C'D'$  in this cube?

m) How many degrees is  $\angle ACC'$  in this cube?

n) How many degrees is  $\angle BDD'$  in this cube?