21. [Angles]

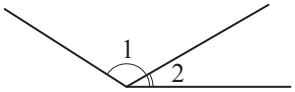
Skill 21.1 Choosing the correct terms related to angles.

- Consider the definitions and properties of a variety of angles. (see Glossary or Math Facts, page 460)

Hints: An angle can be classified according to its size (acute, right, obtuse, straight and reflex). Two angles can be classified according to their position in relation to one another (adjacent, supplementary, complementary or vertical).

Q. Which type of angle describes the pair of angles marked 1 and 2?

A) vertical
B) supplementary
C) adjacent

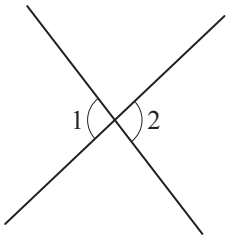


A. *A) vertical ⇒ angles are congruent (1 and 2 are not equal) false*
B) supplementary ⇒ angles add to 180° (1 and 2 add to less than 180°) false
C) adjacent ⇒ angles share the vertex and an arm true

*The answer is **C**.*

a) Which type of angle describes the pair of angles marked 1 and 2?

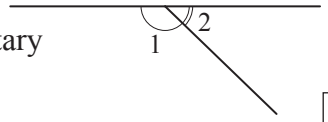
A) right
B) vertical
C) supplementary



B

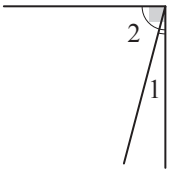
b) Which type of angle describes the pair of angles marked 1 and 2?

A) straight
B) supplementary
C) acute



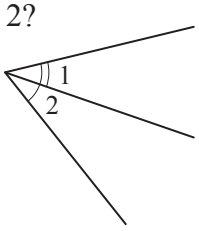
c) Which type of angle describes the pair of angles marked 1 and 2?

A) reflex
B) right
C) complementary



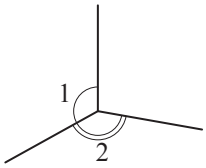
d) Which type of angle describes the pair of angles marked 1 and 2?

A) acute
B) obtuse
C) complementary



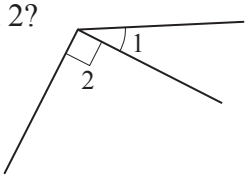
e) Which type of angle describes the pair of angles marked 1 and 2?

A) supplementary
B) obtuse
C) vertical



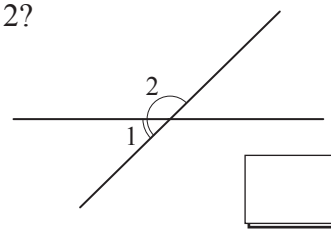
f) Which type of angle describes the pair of angles marked 1 and 2?

A) complementary
B) right
C) adjacent



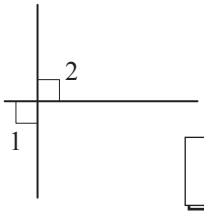
g) Which type of angle describes the pair of angles marked 1 and 2?

A) supplementary
B) acute
C) vertical



h) Which type of angle describes the pair of angles marked 1 and 2?

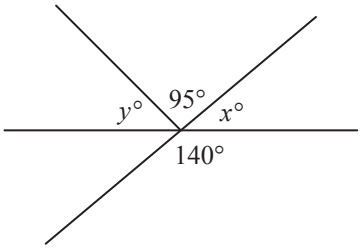
A) straight
B) complementary
C) right



Skill 21.2 Finding the complement and the supplement of a given angle.

- Use the properties:
  - the sum of two complementary angles is 90°
  - the sum of two supplementary angles is 180°.
- Write an equation involving the unknown angle $x°$.
- Solve the equation for $x°$.

Q. Find the values of $x°$ and $y°$.



A. *$x°$ and 140° are supplementary:*

$x° + 140° = 180°$

$x° + 140° - 140° = 180° - 140°$

$x° = \mathbf{40°}$

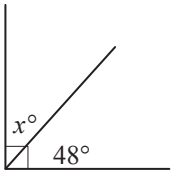
$y°$, 95° and $x°$ are supplementary:

$y° + 95° + 40° = 180°$

$y° + 135° - 135° = 180° - 135°$

$y° = \mathbf{45°}$

a) Find the value of $x°$.

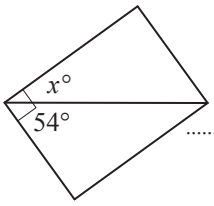


$x° + 48° = 90°$

$x° + 48° - 48° = 90° - 48°$

$x° =$ 42°

b) Find the value of $x°$.

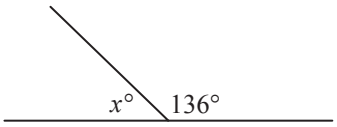


$x° + 54° = 90°$

$=$

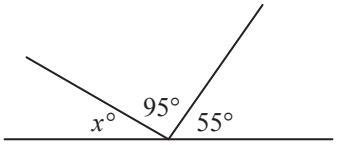
$x° =$

c) Find the value of $x°$.



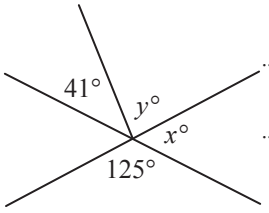
$x° =$

d) Find the value of $x°$.



$x° =$

e) Find the values of $x°$ and $y°$.

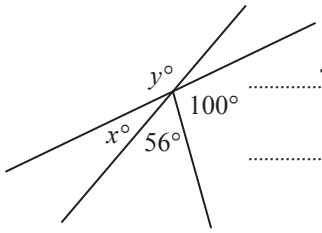


$x° + 125° = 180°$

$x° =$

$y° =$

f) Find the values of $x°$ and $y°$.



$x° + 156° = 180°$

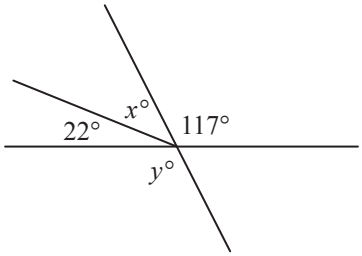
$x° =$

$y° =$

Skill 21.3 Working with vertical angles.

- Use the definition of vertical angles. (see Math Facts, page 460)
- Consider complementary and supplementary angles. (see skill 21.2, page 248)

Q. Find the values of $x°$ and $y°$.



A. *$y°$ and $117°$ are vertical angles:*

$y° = \mathbf{117°}$

$x°$, $22°$ and $117°$ are supplementary:

$x° + 22° + 117° = 180°$

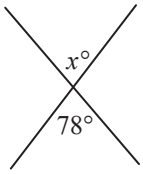
$x° + 139° - 139° = 180° - 139°$

$x° = \mathbf{41°}$

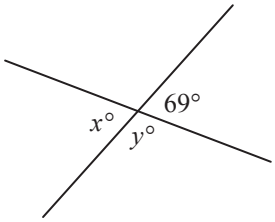
a) Find the value of $x°$.

26°

b) Find the value of $x°$.



c) Find the values of $x°$ and $y°$.



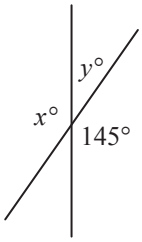
$x° =$

$y° + 69° = 180°$

$y° + 69° - 69° = 180° - 69°$

$y° =$

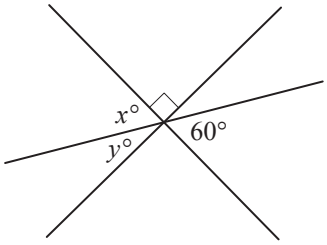
d) Find the values of $x°$ and $y°$.



$x° =$

$y° =$

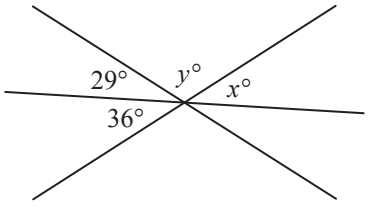
e) Find the values of $x°$ and $y°$.



$x° =$

$y° =$

f) Find the values of $x°$ and $y°$.



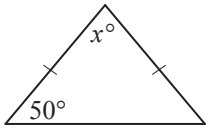
$x° =$

$y° =$

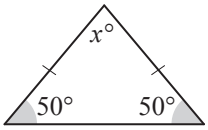
Skill 21.4 Working with angles in a triangle.

- Use the property:
  - the sum of the interior angles of any triangle is 180°.
- Write an equation involving the unknown angle $x°$.
- Solve the equation for $x°$.

Q. Find the value of $x°$.



A.



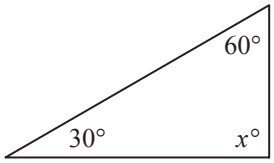
Isosceles triangle ⇒ base angles are equal

$x° + 50° + 50° = 180°$

$x° + 100° - 100° = 180° - 100°$

$x° = \mathbf{80°}$

a) Find the value of $x°$.

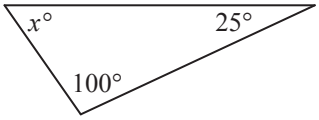


$x° + 30° + 60° = 180°$

$x° + 90° - 90° = 180° - 90°$

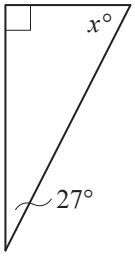
$x° =$ ☐

b) Find the value of $x°$.



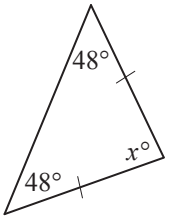
$x° =$ ☐

c) Find the value of $x°$.



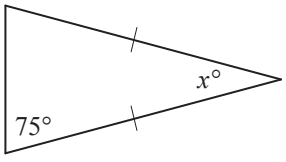
$x° =$ ☐

d) Find the value of $x°$.



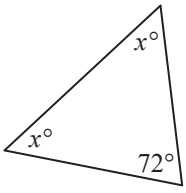
$x° =$ ☐

e) Find the value of $x°$.



$x° =$ ☐

f) Find the value of $x°$.

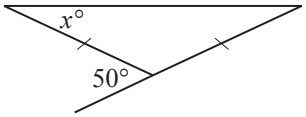


$x° =$ ☐

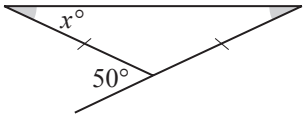
Skill 21.5 Finding the exterior angle of a triangle.

- Use the property:
  - an exterior angle to a triangle is equal to the sum of the two opposite interior angles of the triangle.
- Write an equation involving the unknown angle $x°$.
- Solve the equation for $x°$.

Q. Find the value of $x°$.



A.



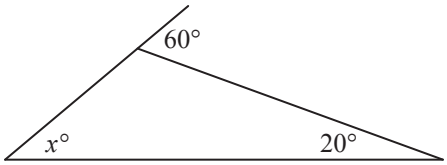
Isosceles triangle ⇒ base angles are equal

$x° + x° = 50°$

$2x° \div 2 = 50° \div 2$

$x° = \mathbf{25°}$

a) Find the value of $x°$.

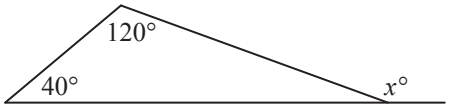


$x° + 20° = 60°$

$x° + 20° - 20° = 60° - 20°$

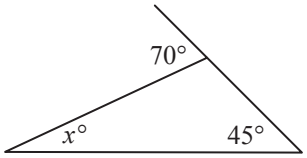
$x° =$ ☐

b) Find the value of $x°$.



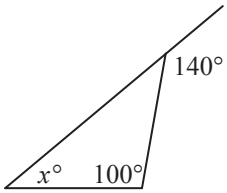
$x° =$ ☐

c) Find the value of $x°$.



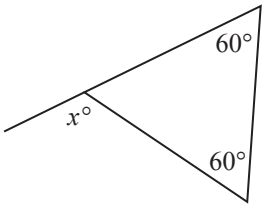
$x° =$ ☐

d) Find the value of $x°$.



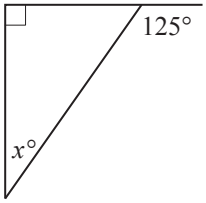
$x° =$ ☐

e) Find the value of $x°$.



$x° =$ ☐

f) Find the value of $x°$.

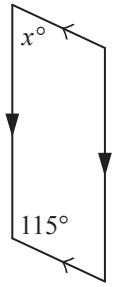


$x° =$ ☐

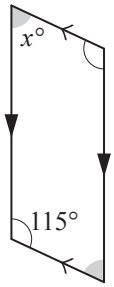
Skill 21.6 Working with angles in a quadrilateral.

- Use the property:
  - the sum of the interior angles of any quadrilateral is 360°.
- Write an equation involving the unknown angle $x°$.
- Solve the equation for $x°$.

Q. Find the value of $x°$.



A.



Parallelogram ⇒ opposite angles are equal

$2x° + 2 \times 115° = 360°$

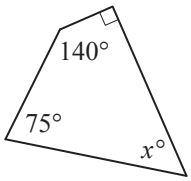
$2x° + 230° - 230° = 360° - 230°$

$2x° = 130°$

$2x° \div 2 = 130° \div 2$

$x° = \mathbf{65°}$

a) Find the value of $x°$.

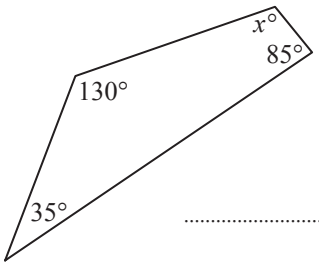


$x° + 90° + 140° + 75° = 360°$

$x° + 305° - 305° = 360° - 305°$

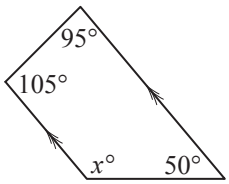
$x° =$ ☐

b) Find the value of $x°$.



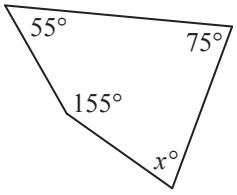
$x° =$ ☐

c) Find the value of $x°$.



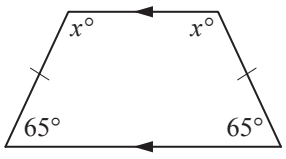
$x° =$ ☐

d) Find the value of $x°$.



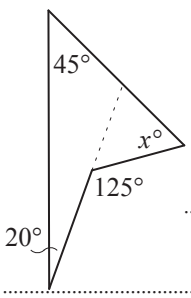
$x° =$ ☐

e) Find the value of $x°$.



$x° =$ ☐

f) Find the value of $x°$.

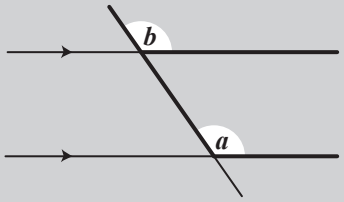


$x° =$ ☐

Skill 21.7 Working with pairs of alternate interior, same-side interior and corresponding angles.

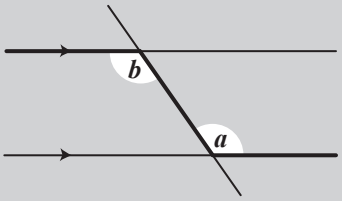
- Consider the classification and properties of the angles formed by intersecting a pair of parallel lines by a transversal. (see Math Facts, page 460)

CORRESPONDING



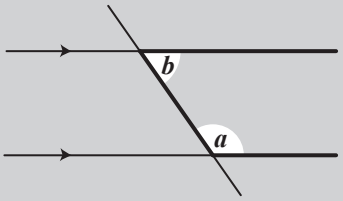
$\angle a = \angle b$

ALTERNATE INTERIOR



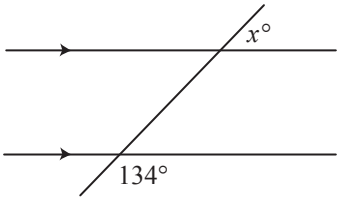
$\angle a = \angle b$

SAME-SIDE INTERIOR

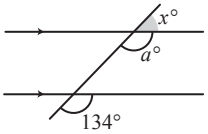


$\angle a + \angle b = 180°$

Q. Find the value of $x°$.



A.



x° and a° are not in any category above

a° and 134° are corresponding angles

$\Rightarrow a° = 134°$

x° and a° are supplementary angles

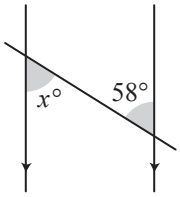
$\Rightarrow x° + a° = 180°$

Substitute $a° = 134° \Rightarrow x° + 134° = 180°$

$x° + 134° - 134° = 180° - 134°$

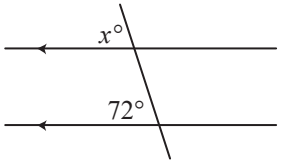
$x° = \mathbf{46°}$

a) Find the value of $x°$.



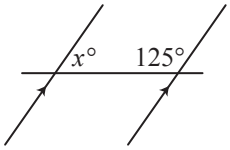
alternate interior angles $\Rightarrow x° =$ .....

b) Find the value of $x°$.



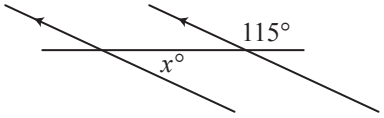
$\Rightarrow x° =$ .....

c) Find the value of $x°$.



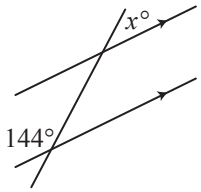
$\Rightarrow x° =$ .....

d) Find the value of $x°$.



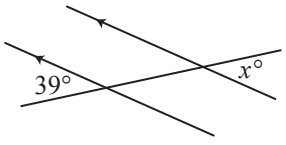
$\Rightarrow x° =$ .....

e) Find the value of $x°$.



$\Rightarrow x° =$ .....

f) Find the value of $x°$.

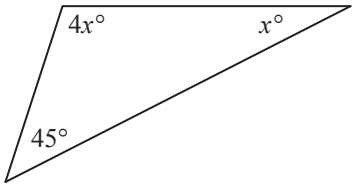


$\Rightarrow x° =$ .....

Skill 21.8 Finding the value of an angle in a variety of diagrams.

- Use the properties of angles. (see skills 21.1 to 21.7, pages 247 to 253 and Math Facts, page 460)
- Write an equation involving the unknown angle $x°$.
- Solve the equation for $x°$.

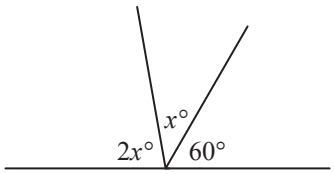
Q. Find the value of $x°$.



A.

$$4x° + x° + 45° = 180°$$
$$5x° + 45° - 45° = 180° - 45°$$
$$5x° = 135°$$
$$5x° \div 5° = 135° \div 5°$$
$$x° = \mathbf{27°}$$

a) Find the value of $x°$.

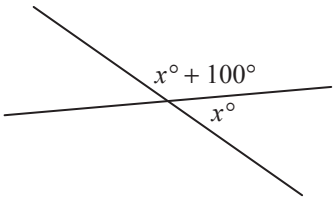


$3x° + 60° - 60° = 180° - 60°$

$3x° \div 3 = 120° \div 3$

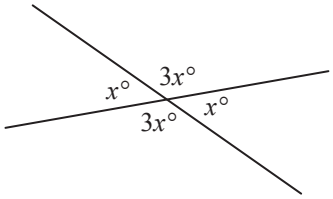
$x° =$

b) Find the value of $x°$.



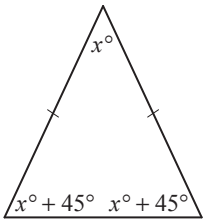
$x° =$

c) Find the value of $x°$.



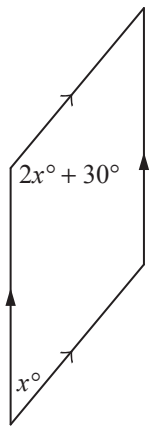
$x° =$

d) Find the value of $x°$.



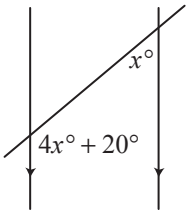
$x° =$

e) Find the value of $x°$.



$x° =$

f) Find the value of $x°$.



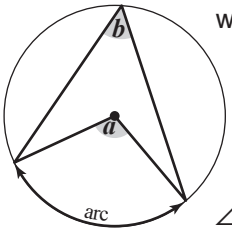
$x° =$

continues on page 256

Skill 21.9 Finding the value of an angle in a circle (1).

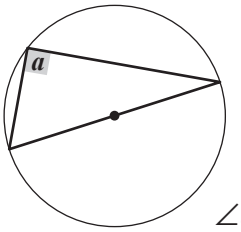
- Use the following properties of angles in circles:

Property 1 The angle at the center of a circle is twice the size of the inscribed angle which intercepts the same arc of the circle.



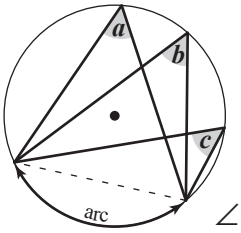
$\angle a = 2 \times \angle b$

Property 2 An angle inscribed in a semicircle is a right angle.



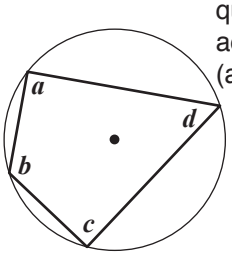
$\angle a = 90°$

Property 3 All inscribed angles that intercept the same arc of the circle are equal.



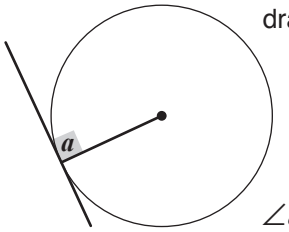
$\angle a = \angle b = \angle c$

Property 4 The opposite angles in a quadrilateral inscribed in a circle add up to 180° (are supplementary).



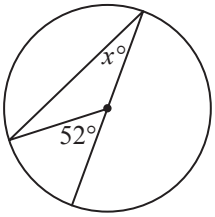
$\angle a + \angle c = 180°$
$\angle b + \angle d = 180°$

Property 5 If a line is tangent to a circle, then the line is perpendicular to the radius drawn to the point of tangency.



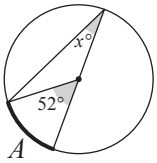
$\angle a = 90°$

Q. Find the value of $x°$.



A.

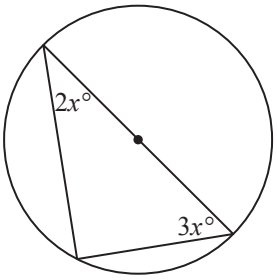
use property 1



52° is an angle at the center intercepting arc A
$x°$ is an inscribed angle intercepting arc A $\Rightarrow 2x° = 52°$
Solve the equation:

$2x° = 52°$
$2x° \div 2 = 52° \div 2$
$x° = \mathbf{26°}$

a) Find the value of $x°$.



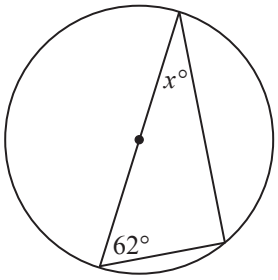
use property 2

$2x° + 3x° = 90°$

$5x° \div 5 = 90° \div 5$

$x° =$ ☐

b) Find the value of $x°$.

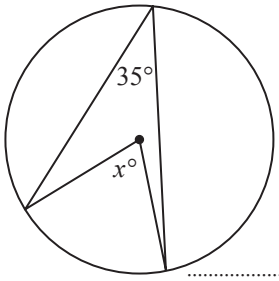


$x° =$ ☐

continued from page 255

Skill 21.9 Finding the value of an angle in a circle (2).

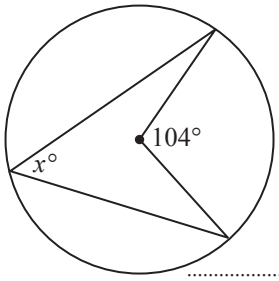
c) Find the value of $x°$.



$x° = 2 \times 35°$

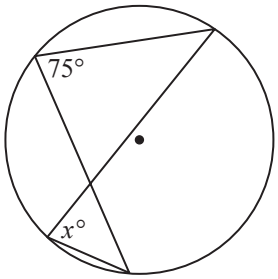
$x° =$ ☐

d) Find the value of $x°$.



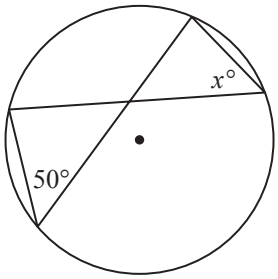
$x° =$ ☐

e) Find the value of $x°$.



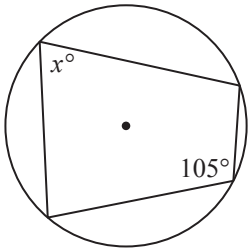
$x° =$ ☐

f) Find the value of $x°$.



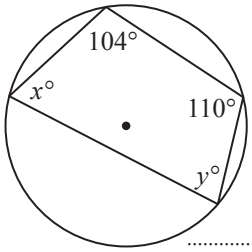
$x° =$ ☐

g) Find the value of $x°$.



$x° =$ ☐

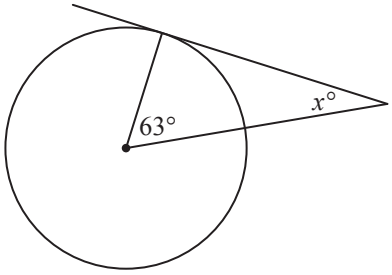
h) Find the values of $x°$ and $y°$.



$x° =$ ☐

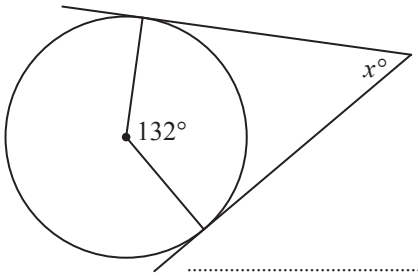
$y° =$ ☐

i) Find the value of $x°$.



$x° =$ ☐

j) Find the value of $x°$.



$x° =$ ☐