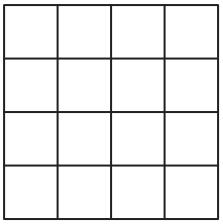
22. [Exploring Geometry]

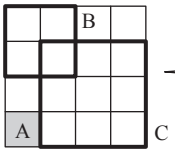
Skill 22.1 Naming and labeling geometric plane shapes.

- Label, with capital letters, any plane geometric shape by starting at one vertex and moving to the next adjacent vertex, without skipping any vertex.
- Use a color code to count all the different types of shapes inside a diagram, or count by size.

Q. How many different squares are there in this diagram?



A.



Count the squares according to side length

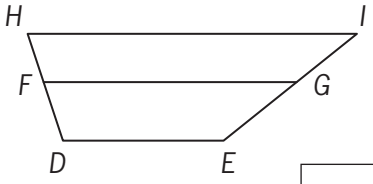
original square

(A) *16* + (B) *9* + (C) *4* + *1* = *30*

*There are **30** different squares in the diagram.*

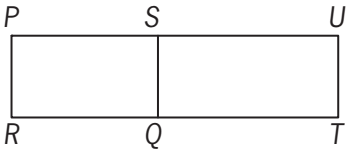
a) Which label names a trapezoid drawn in this diagram?

A) *DEGH*
B) *DHIG*
C) *FHIG*
D) *GIDE*



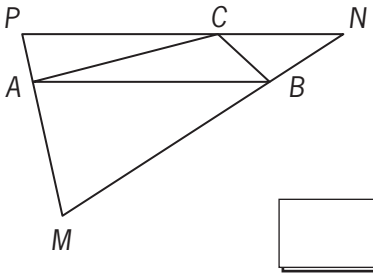
b) Which label names a rectangle drawn in this diagram?

A) *TRSU*
B) *QSUR*
C) *PRUT*
D) *PRQS*



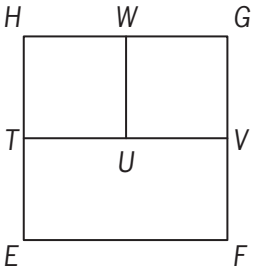
c) Which label names a triangle drawn in this diagram?

A) *PBM*
B) *CAM*
C) *BNC*
D) *BCM*

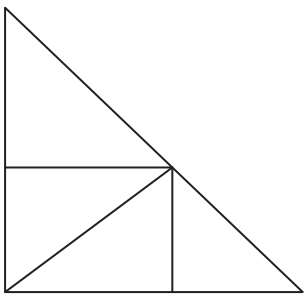


d) Which label names a rectangle drawn in this diagram?

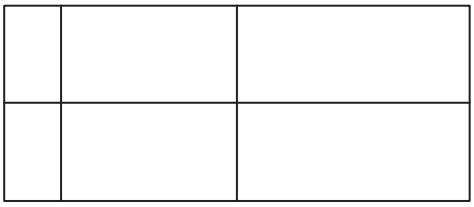
A) *HUWT*
B) *TEFV*
C) *TVEF*
D) *HWVT*



e) How many different triangles are there in this diagram?



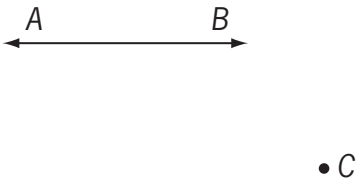
f) How many different rectangles are there in this diagram?



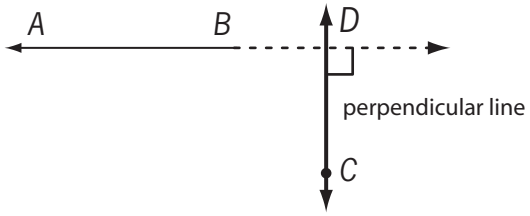
Skill 22.2 Sketching parallel and perpendicular lines to given lines.

- See the definitions of altitude, parallel and perpendicular lines. (see Glossary)

Q. Sketch the line $\overleftrightarrow{CD}$ which is perpendicular to $\overleftrightarrow{AB}$ and passes through point C.

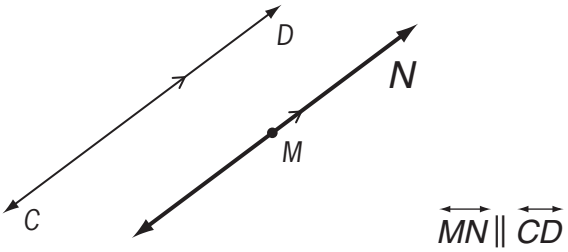


A. *The perpendicular line from point C is the line that starts from C and makes a right angle with $\overleftrightarrow{AB}$.*



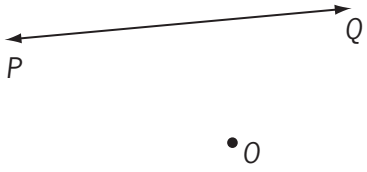
In this case, the perpendicular from point C intersects the extension of $\overline{AB}$ in D: $\overleftrightarrow{CD} \perp \overleftrightarrow{AB}$ It cannot land inside $\overline{AB}$, because the angle between $\overleftrightarrow{AB}$ and the perpendicular will not be 90°.

a) Sketch the line $\overleftrightarrow{MN}$ which runs parallel to $\overleftrightarrow{CD}$ and passes through point M.

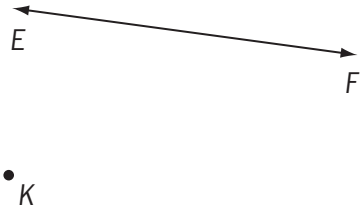


$\overleftrightarrow{MN} \parallel \overleftrightarrow{CD}$

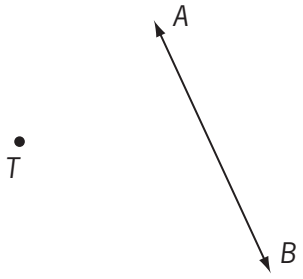
b) Sketch the line $\overleftrightarrow{OR}$ which is perpendicular to $\overleftrightarrow{PQ}$ and passes through point O.



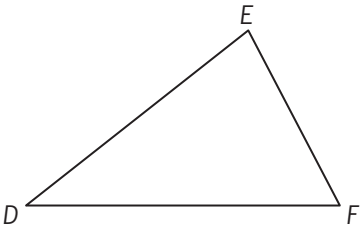
c) Sketch the line $\overleftrightarrow{KL}$ which runs parallel to $\overleftrightarrow{EF}$ and passes through point K.



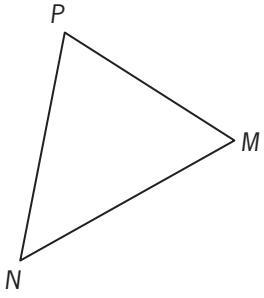
d) Sketch the line $\overleftrightarrow{TU}$ which is perpendicular to $\overleftrightarrow{AB}$ and passes through point T.



e) Sketch the altitude $\overleftrightarrow{DD'}$ in this triangle.



f) Sketch the altitude $\overleftrightarrow{MM'}$ in this triangle.



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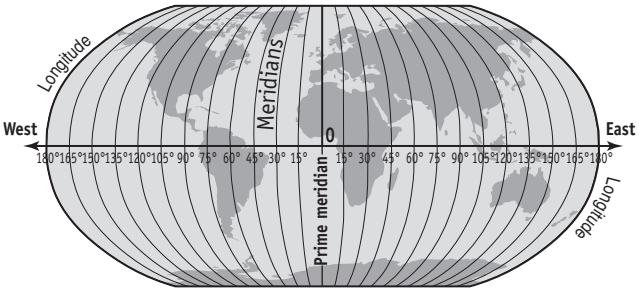
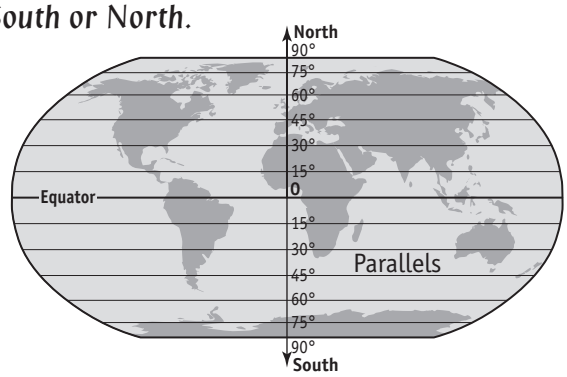
Skill 22.3 Describing location on a map (1).

- On a geographic map:
  - follow the horizontal latitude and vertical longitude lines to describe the location of a point.

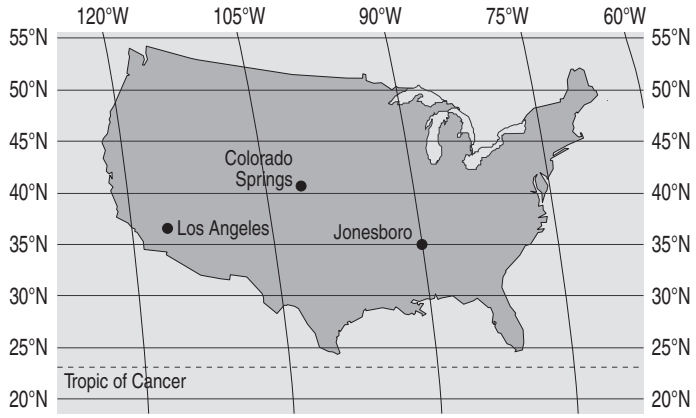
Hints: The vertical longitude lines on a map are called meridians.
Longitude is usually given in degrees and minutes East or West.
The horizontal latitude lines on a map are called parallels.
Latitude is usually given in degrees and minutes South or North.





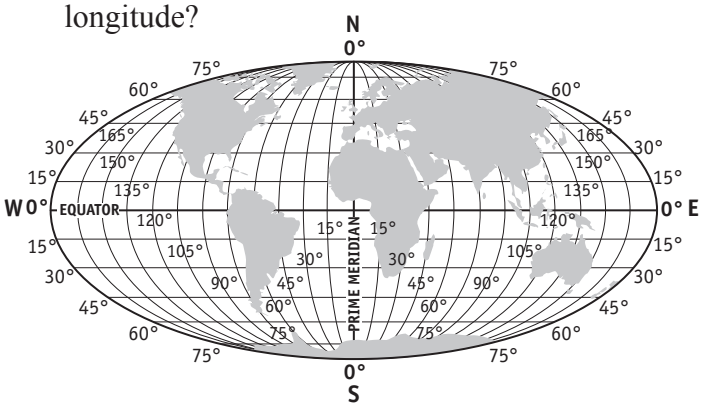
- On a weather map:
  - read the value indicated on an isobar to describe the location of any point situated on or inside that isobar.

Q. To the nearest five degrees, what is the latitude and longitude of Jonesboro, Arkansas?

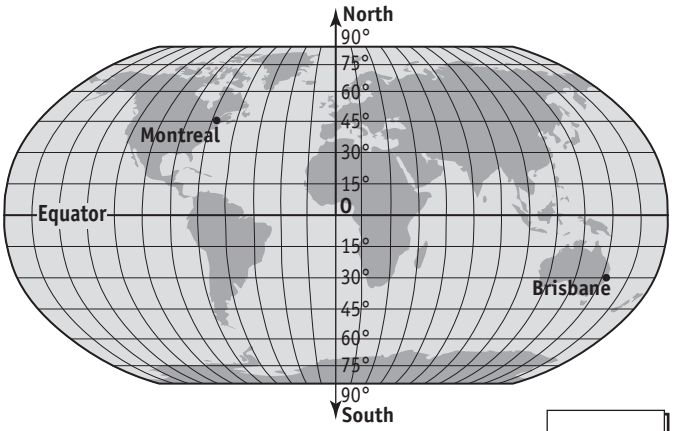


A. *The parallel (horizontal line) closest to Jonesboro represents 35° north latitude.*
Jonesboro is located on the 90° west longitude line (meridian).
To the nearest five degrees, Jonesboro is located at ***35°N latitude*** *and* ***90°W longitude****.*

a) Which continent would you be in if you are located at 75° south latitude and 40° east longitude?



b) How many degrees latitude are between Montreal (Canada) and Brisbane (Australia)?



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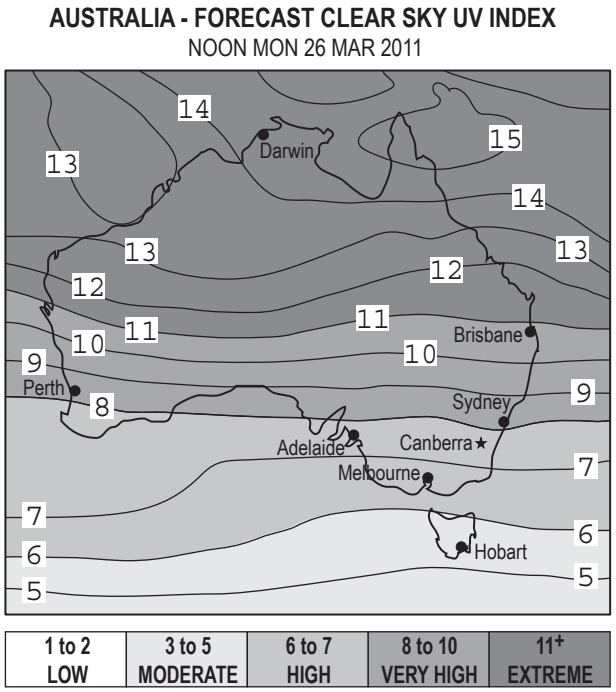
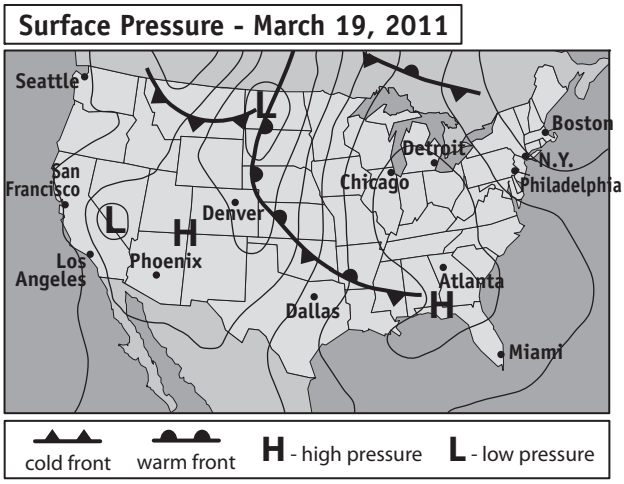
Skill 22.3 Describing location on a map (2).

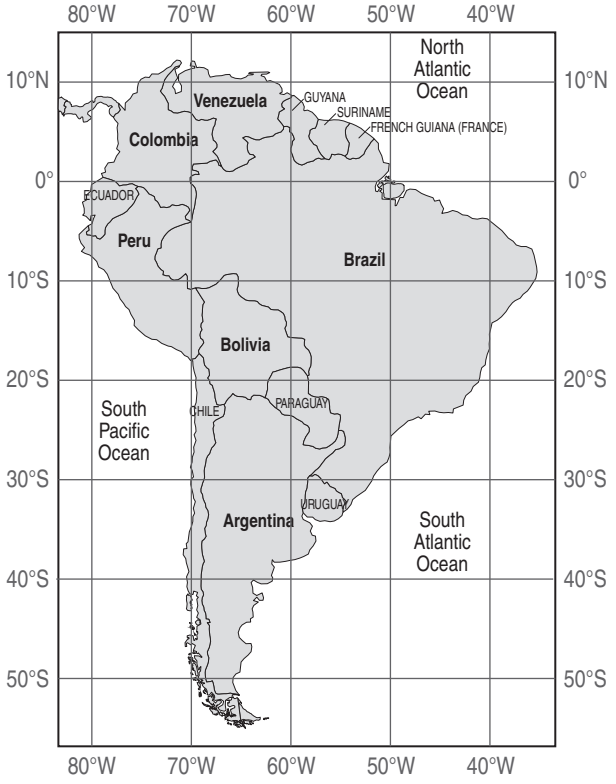
c) How can the UV index be described for Canberra at noon on the 26th March 2011?



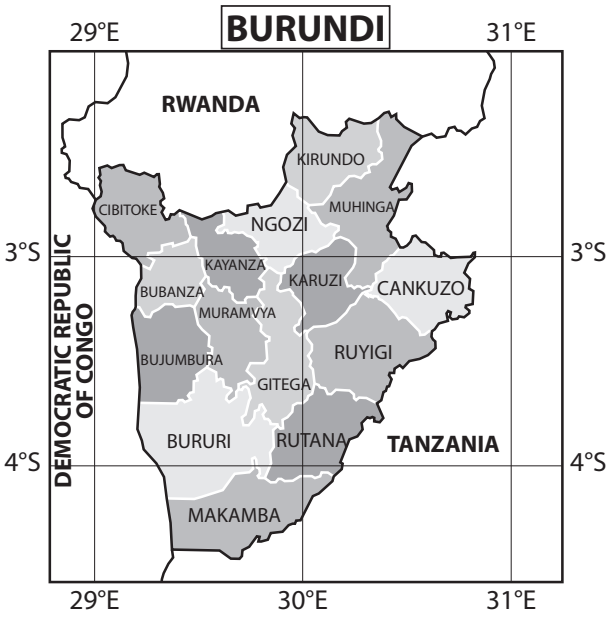
d) What kind of surface pressure is shown above Detroit on this map?



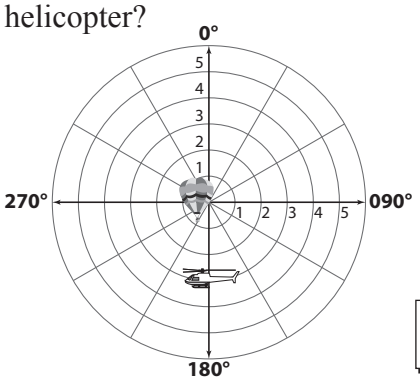
e) Of the lines of longitude shown, between which two lines is Bolivia mainly situated?



f) Name the province in Burundi where the 4°S parallel intersects the 30°E meridian.



g) On this radar screen, what is the altitude of the helicopter?

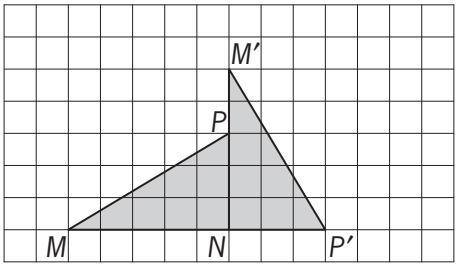


m

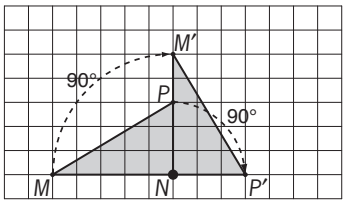
Skill 22.4 Recognizing basic transformations of two-dimensional shapes.

- Use the definitions of reflection, rotation and translation. (see Glossary)

Q. Which transformation (translation, reflection or rotation) has moved triangle *MNP*?



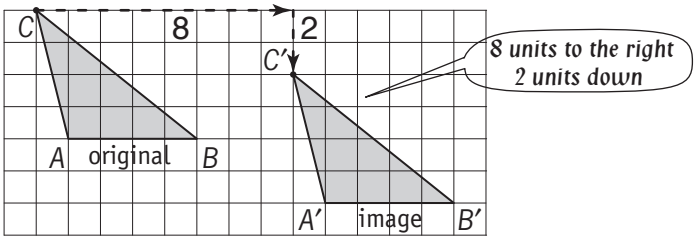
A.



N does not move.
M rotates 90° clockwise about point N.
P rotates 90° clockwise about point N.

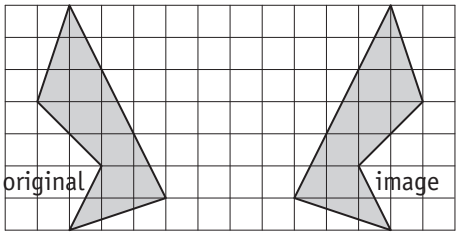
The transformation is a ***rotation****.*

a) Which transformation (translation, reflection or rotation) has moved triangle *ABC*?

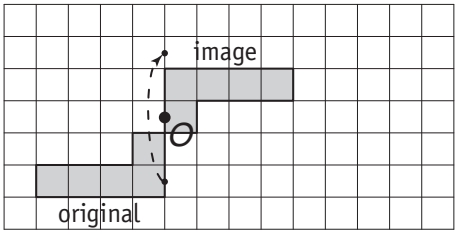


translation

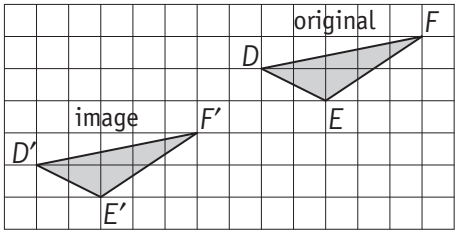
b) Which transformation (translation, reflection or rotation) has moved this shape?



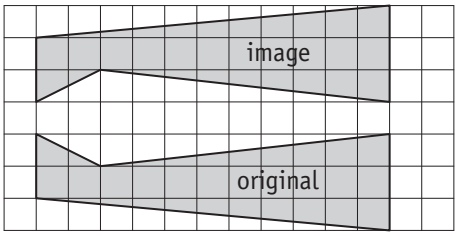
c) Which transformation (translation, reflection or rotation) has moved this shape?



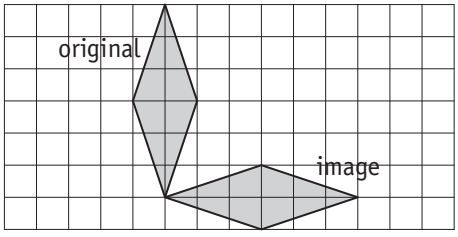
d) Which transformation (translation, reflection or rotation) has moved triangle *DEF*?



e) Which transformation has moved this shape?



f) Which transformation has moved this shape?



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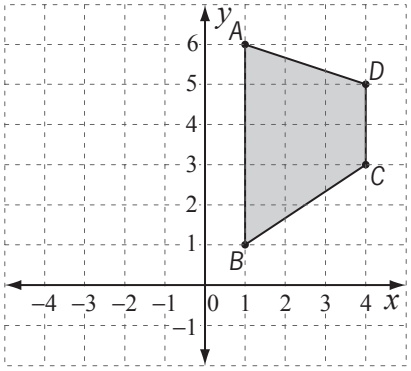
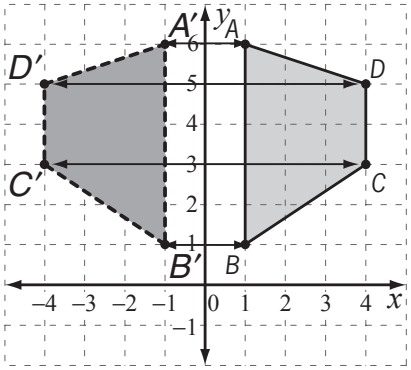
Skill 22.5 Drawing translations, reflections and rotations on a grid or coordinate plane (1).

- To draw a shape translated on a grid:
  move it up (positive, vertically), down (negative, vertically), left (negative, horizontally) or right (positive, horizontally) on the grid, without flipping, turning or changing its size.
- To draw a shape reflected in a given line (mirror line):
  - draw a perpendicular line to the mirror line from each vertex of the shape
  - extend the perpendicular line beyond the mirror line by the same distance
  - plot and join the reflected points.
- To draw a shape rotated by a given angle about a point (center of rotation):
  - rotate each vertex by the given angle, in the given direction
  - plot and join the rotated points.

Q. Draw the reflection of the trapezoid *ABCD* in the line of equation $x = 0$

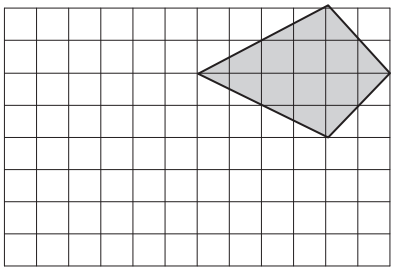


A. *Reflect A, B, C and D in the mirror line $x = 0$:*
A is one unit to the right of line $x = 0$
$\Rightarrow$ *draw A′ one unit to the left of line $x = 0$.*

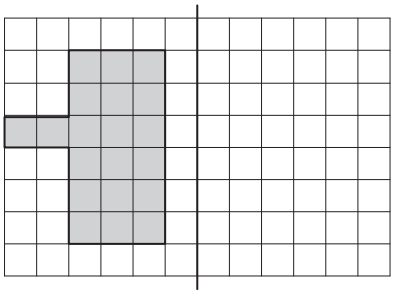


A′B′C′D′ *is the reflection of ABCD in the line $x = 0$*

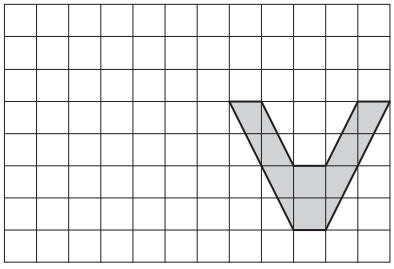
a) Redraw this shape translated 6 units to the left and 4 units down.



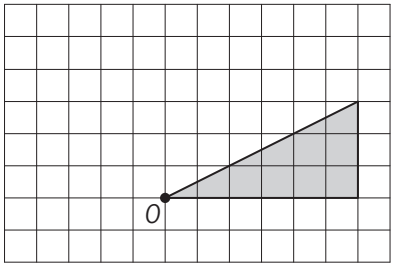
b) Redraw this shape reflected in the line shown.



c) Redraw this shape translated 7 units to the left and 1 unit up.



d) Redraw this triangle rotated 90° about point *O* in an anticlockwise direction.



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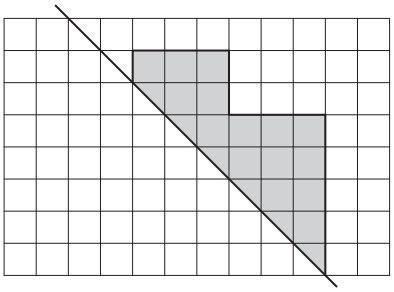
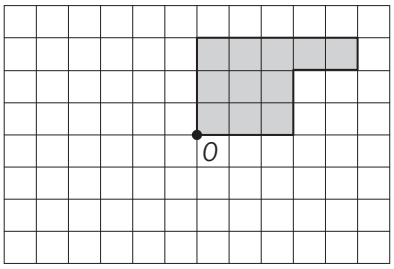
Skill 22.5 Drawing translations, reflections and rotations on a grid or coordinate plane (1).

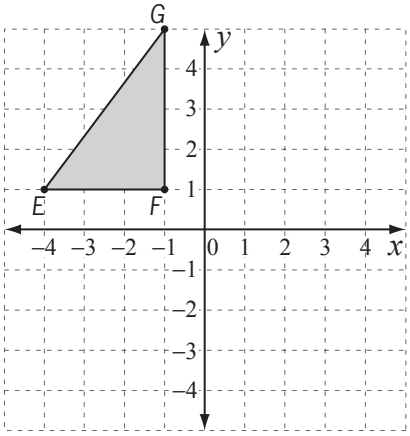
e) Redraw this shape reflected in the line shown.



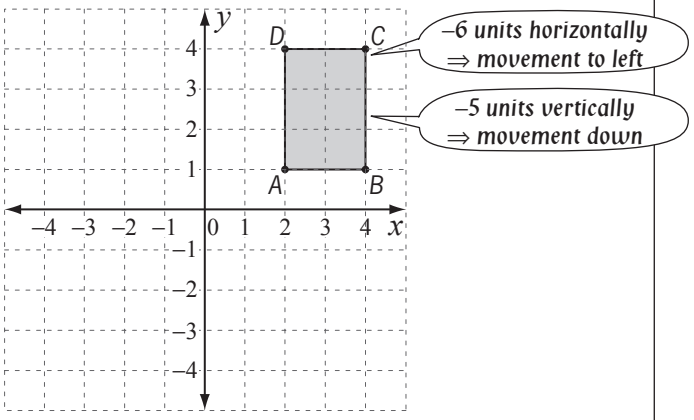
f) Redraw this shape rotated 180° about point O in a clockwise direction.



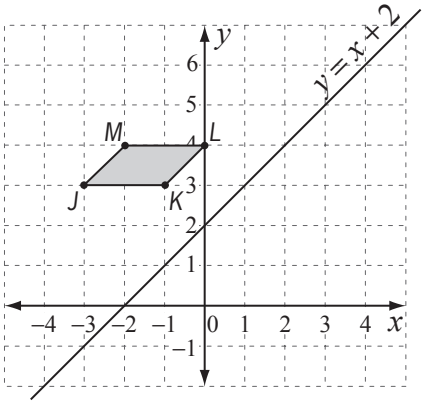
g) Redraw the triangle EFG rotated about point F by 180°.



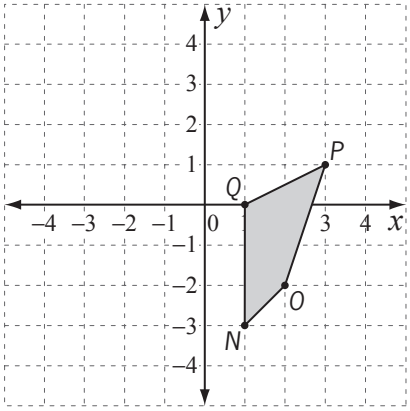
h) Redraw the rectangle $ABCD$ after translating it −6 units horizontally and −5 units vertically.



i) Draw the reflection of the parallelogram $JKLM$ in the line of equation $y = x + 2$



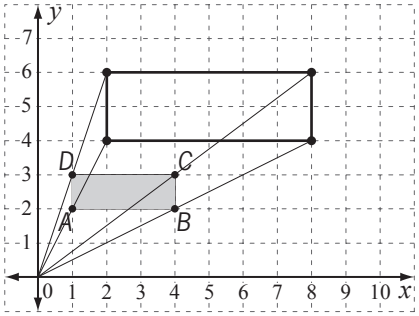
j) Redraw the quadrilateral $NOPQ$ rotated about point Q by 180° in an anticlockwise direction.



Skill 22.6 Recognizing and drawing enlargements and reductions on a grid or a coordinate plane.

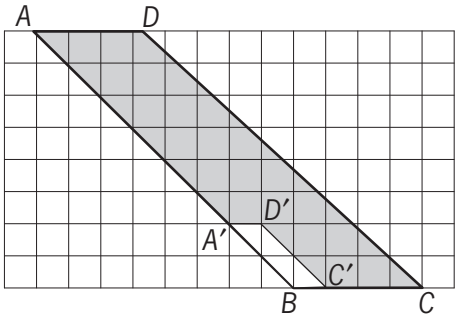
- Use the definitions of enlargement, reduction, factor of enlargement and factor of reduction. (see Glossary)

Q. Find the scale factor of enlargement for the rectangle *ABCD*.

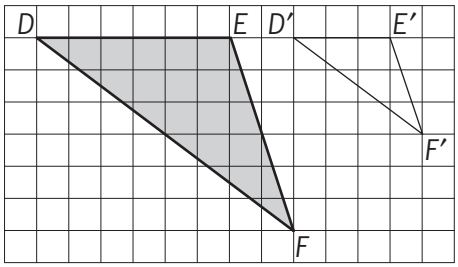


A. *The length and width of the rectangle ABCD have doubled in the enlargement.*
Scale factor of enlargement = ***2***

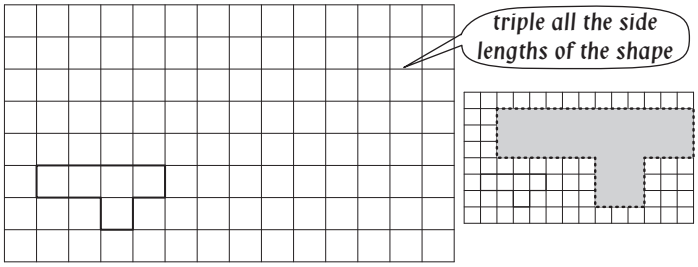
a) Find the scale factor of reduction for the parallelogram *ABCD*.



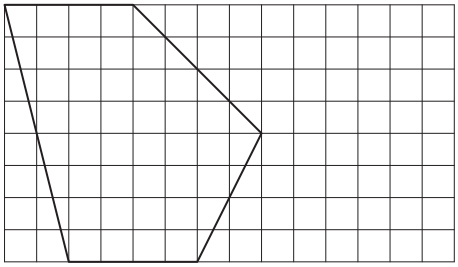
b) Find the scale factor of reduction for the triangle *DEF*.



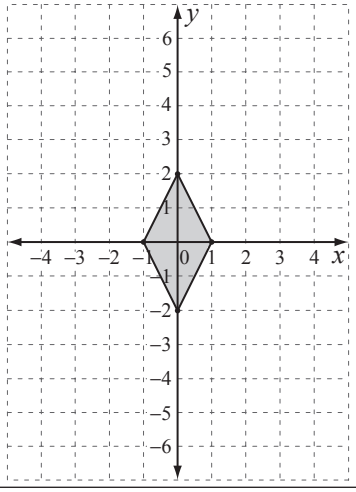
c) Redraw this shape enlarged by a factor of 3.



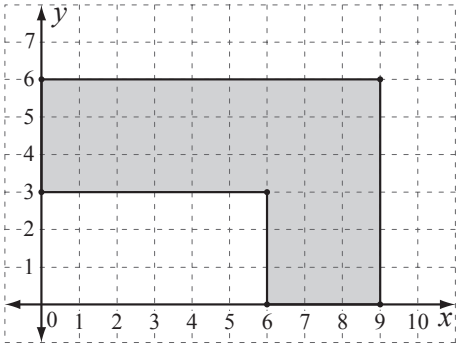
d) Redraw this shape reduced by a factor of 2.



e) Redraw this rhombus enlarged by a scale factor of 3 about the origin of the axes.



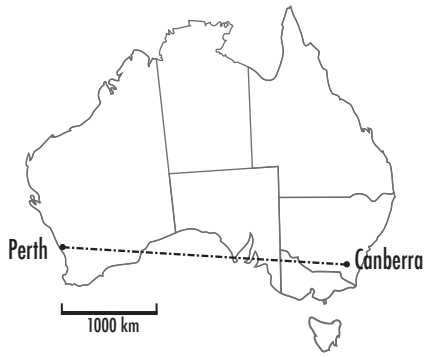
f) Redraw this shape reduced by a scale factor of 3 about the origin of the axes.



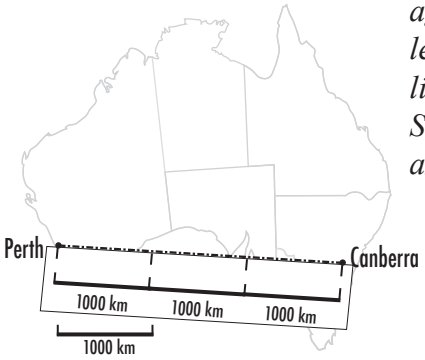
Skill 22.7 Working with scales on a map.

- Put a piece of paper (or ruler) along the distance to be measured.
- Mark the start and end points on the paper.
- Place the paper against the scale matching the starting points.
- Slide the paper across the length of the scale marking the start and end points as you go.
- Add together the scale lengths covered.

Q. Using the scale below, what is the marked distance from Perth to Canberra?

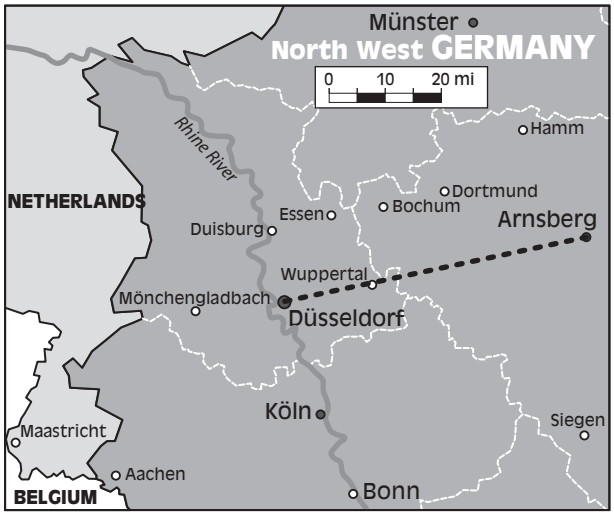


A. $3 \times 1000 =$ ***3000 km***



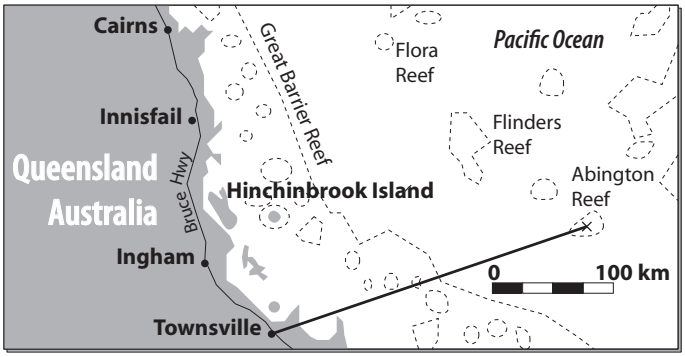
Check the scale against the length of the line. Slide the scale as necessary.

a) Use the scale to find the distance from Düsseldorf to Arnsberg.



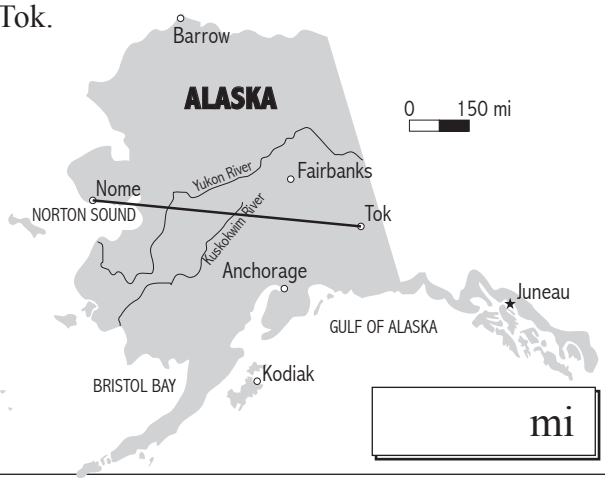
mi

b) Use the scale to find the distance from Townsville to Abington Reef.



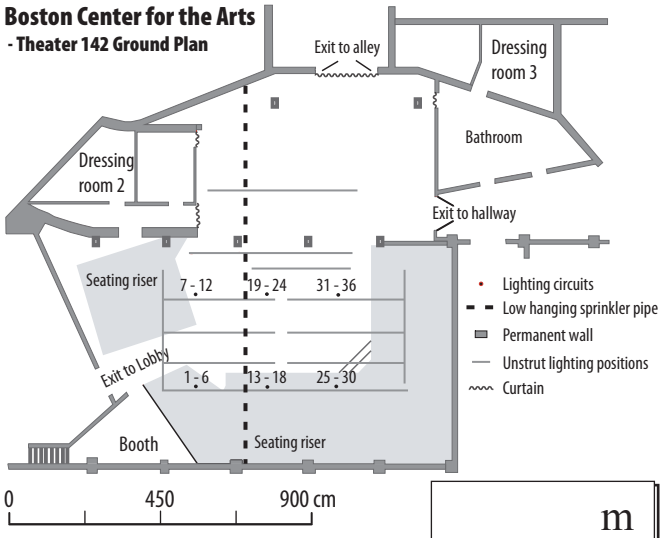
km

c) Use the scale to find the distance from Nome to Tok.



mi

d) What is the length of the low hanging sprinkler pipe at Boston Center for the Arts?

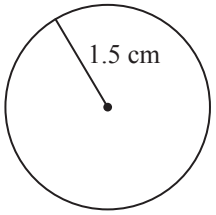


m

Skill 22.8 Finding the scale factor of a model.

- Convert the real-life dimension to the same unit of measurement of the model dimension.
- Divide the real-life dimension by the model dimension, to find the scale factor.
- Write the scale ratio of the model in the form of **1 : scale factor**.

Q. Determine the scale factor used when this circle represents the plan of a lake of diameter 600 m.



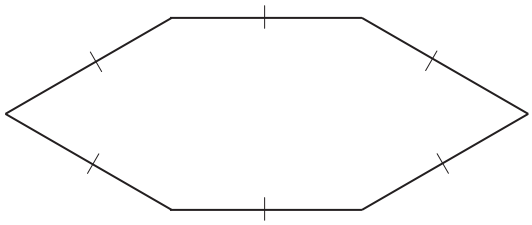
A. $600\ m = 600 \times 100\ cm$ (1 m = 100 cm)
$= 60{,}000\ cm$

$scale\ factor = 60{,}000\ cm \div 3\ cm$
$= \mathbf{20{,}000}$

diameter $= 2 \times radius$
$= 2 \times 1.5$
$= 3$

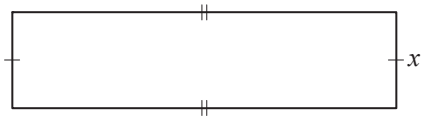
$scale\ ratio = 1 : \mathbf{20{,}000}$

a) A scale of 1 in. = 13 ft is used to draw this plan of a garden bed. Using your ruler and the scale, what is the real distance around the garden bed?



..... = ft

b) A scale of 1 in. = 15 yd is used to draw this plan of the yard. Using your ruler and the scale, what is the real distance marked x?

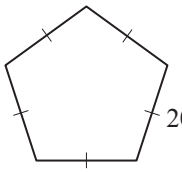


.....

.....

..... = yd

c) Determine the scale factor used when this regular pentagon represents the plan of a bike track of perimeter 6 km.



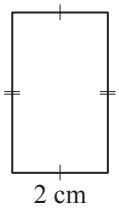
.....

.....

scale factor = ...

..... = 1 :

d) Determine the scale factor used when this rectangle represents the plan of a football stadium of width 60 m.



.....

.....

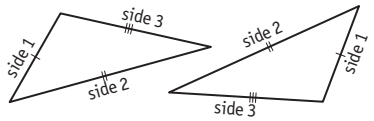
scale factor = ...

..... = 1 :

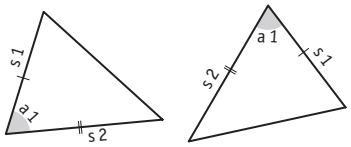
Skill 22.9 Recognizing congruence of two-dimensional shapes.

- Use the **postulates for congruence** to check if two shapes are congruent (same size and shape).

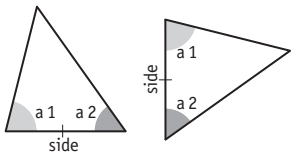
1. Side-side-side (**SSS**)



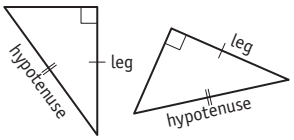
2. Side-angle-side (**SAS**)



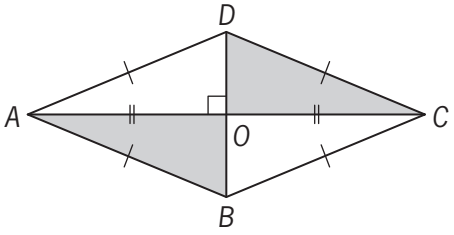
3. Angle-side-angle (**ASA**)



4. Hypotenuse-leg (**HL**)

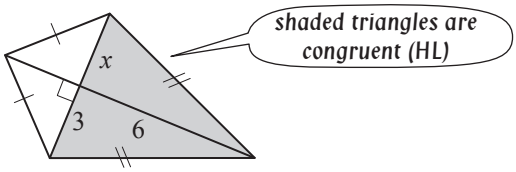


Q. Which congruence postulate (SSS, SAS, ASA, HL) can be applied to show that triangle *AOB* is congruent to triangle *COD*?

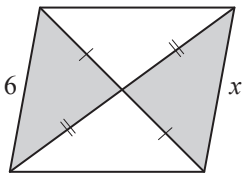


A. $\overline{AB} \cong \overline{CD}$ *(hypotenuse)*
$\overline{AO} \cong \overline{OC}$ *(leg)*
$\Rightarrow \Delta AOB \cong \Delta COD$
These triangles are congruent based on the congruence postulate hypotenuse-leg, ***HL***

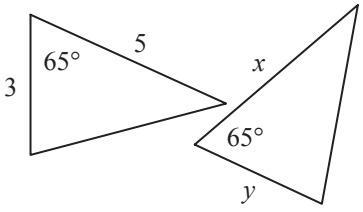
a) Find the value of x given the pair of shaded triangles are congruent.



b) Find the value of x given the pair of shaded triangles are congruent.

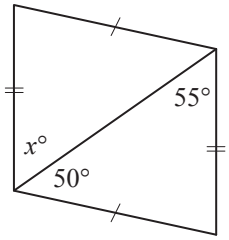


c) Find the values of x and y in this pair of congruent triangles.

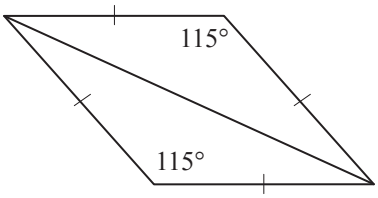


$x =$ $y =$

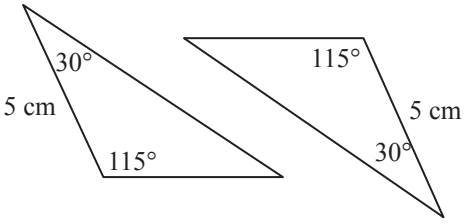
d) Find the value of $x°$ in this pair of congruent triangles.



e) Which congruence postulate (SSS, SAS, ASA) can be applied to show that these triangles are congruent?



f) Which congruence postulate (SSS, SAS, ASA) can be applied to show that these triangles are congruent?



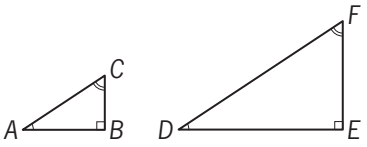
Skill 22.10 Recognizing similarity of two-dimensional shapes.

To check if two shapes are similar (same shape, but different size):
EITHER

- Measure the size of the angles. Two shapes are similar if they have corresponding angles the same size.

OR

- Measure the size of the sides. Two shapes are similar if they have corresponding sides proportional (each pair of corresponding sides is in the same ratio as every other pair).



Corresponding angles are congruent:

$\angle A \cong \angle D$ and $\angle B \cong \angle E$ and $\angle C \cong \angle F$

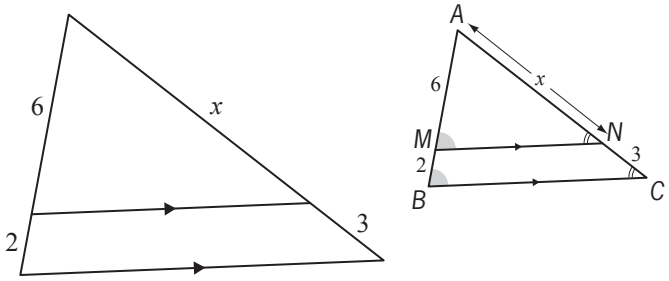
Corresponding sides are proportional:

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

To find the value of an unknown side:

- Locate two pairs of corresponding sides; one pair should involve the unknown side.
- Set up the proportion (two equal ratios written as an equation).
- Solve the equation to find the value of the unknown.

Q. Find the value of x. [All measurements are in cm.]

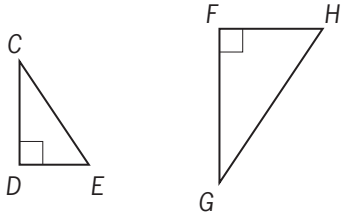


A. *Triangles AMN and ABC are similar because:*
$\angle MAN \cong \angle BAC$ (the same angle)
$\angle AMN \cong \angle ABC$ (corresponding angles)
$\angle ANM \cong \angle ACB$ (corresponding angles)
The sides are proportional:

$$\frac{AM}{AB} = \frac{AN}{AC} \Rightarrow \frac{6}{8} = \frac{x}{x+3}$$ Cross multiply

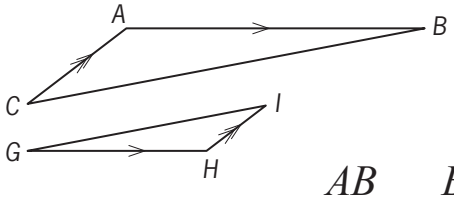
$$6x + 18 = 8x$$
$$6x + 18 - 6x = 8x - 6x$$ Inverse of $+6x$ is $-6x$
$$2x = 18$$
$$x = \mathbf{9}$$

a) Complete the pairs of congruent angles for these similar triangles.



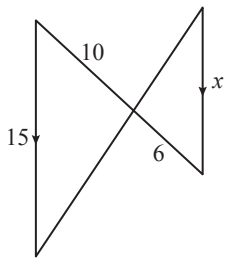
$\angle C \cong$, $\angle D \cong$, $\angle E \cong$

b) Complete the ratios of corresponding sides for these similar triangles.



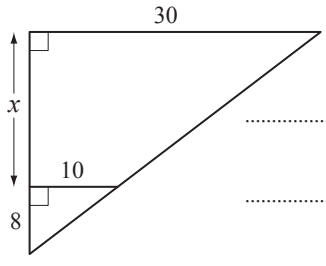
$$\frac{AB}{\square} = \frac{BC}{\square} = \frac{AC}{\square}$$

c) Find the value of x. [All measurements are in cm.]



$x =$

d) Find the value of x. [All measurements are in cm.]

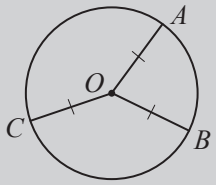


$x =$

Skill 22.11 Recognizing elements of circle geometry.

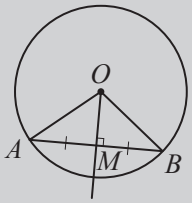
- Consider the properties of a segment in a circle.

Radius -The radii in a circle are the same length.



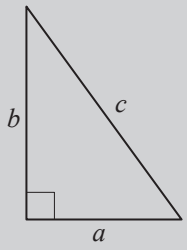
$\overline{OA} \cong \overline{OB} \cong \overline{OC}$

Chord - In a circle, a diameter that is perpendicular to a chord bisects the chord.



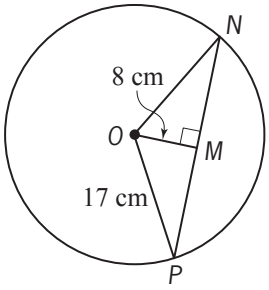
$\overline{AM} \cong \overline{MB}$

Pythagorean Theorem



$a^2 + b^2 = c^2$

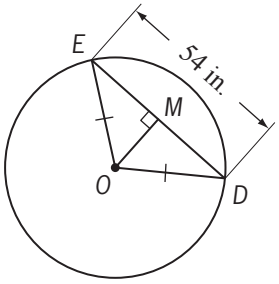
Q. A chord $\overline{NP}$ is 8 cm from the center of a circle of radius 17 cm. Find the length of chord $\overline{NP}$.
[Pythagorean theorem will help.]



A. *Apply Pythagorean Theorem in ΔOMP:*

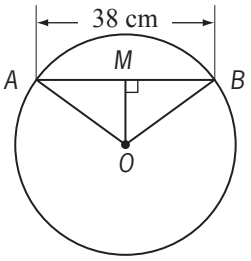
$OP^2 = OM^2 + MP^2$ (find the length MP first)
$17^2 = 8^2 + MP^2$
$MP^2 = 289 - 64$
$MP^2 = 225$
$MP = 15$
$\Rightarrow NP = 2 \times MP$ (MP = MN = 15 cm)
$= 2 \times 15$
$= \mathbf{30\ cm}$

a) Find the distance DM.



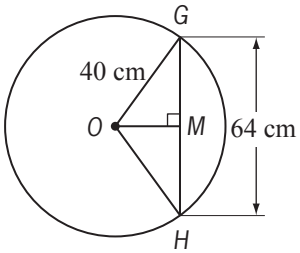
in.

b) Find the distance BM.



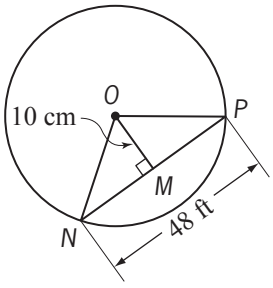
cm

c) Find the length of the segment $\overline{OM}$.
[Pythagorean theorem will help.]



$OM =$ ..... $=$ cm

d) Find the length of the radius of the circle.
[Pythagorean theorem will help.]



..... $=$ ft