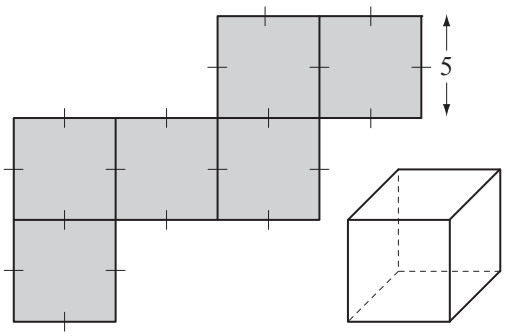
26. [Surface Area]

continues on page 306

Skill 26.1 Calculating the surface area of rectangular prisms and cubes by using nets (1).

- Find any unknown side lengths.
- Calculate the area of each face as shown on the net.
  Hint: Rectangular prisms have 6 faces of 3 different sizes: base and top (2)
  front and back (2)
  other faces (2)
- Add together the area of all faces.
  Hints: Sides marked with a dash (|) are of equal length.
  Sides marked with two dashes (||) are of equal length etc.

Q. Find the surface area of the cube by finding the area of its net.

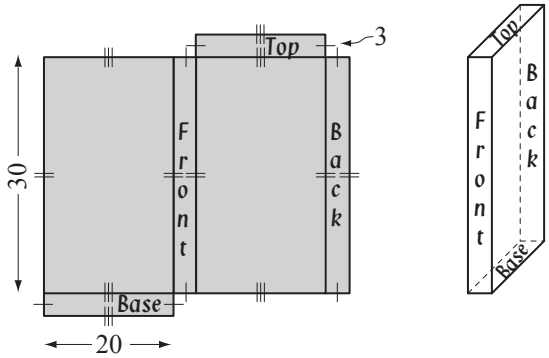


A. *Area of square face* $= 5 \text{ units} \times 5 \text{ units}$
$= 25 \text{ sq. units}$

$S.A. = 25 \times 6$
$= \mathbf{150 \text{ sq. units}}$

A cube has 6 identical faces

a) Find the surface area of the rectangular prism by finding the area of its net.



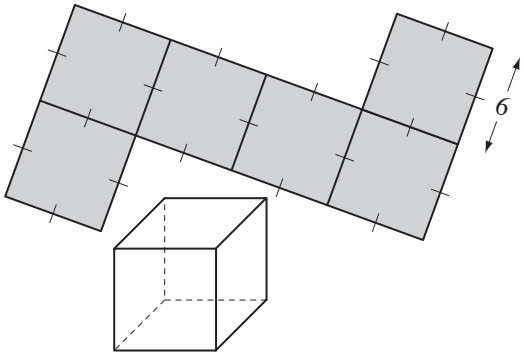
Area: base & top $= 2 \times 20 \times 3 = 120$

Area: front & back $= 2 \times 30 \times 3 = 180$

Area: 2 other faces $= 2 \times 30 \times 20 = 1200$

S.A. $= 120 + 180 + 1200 =$ sq. units

b) Find the surface area of the cube by finding the area of its net.



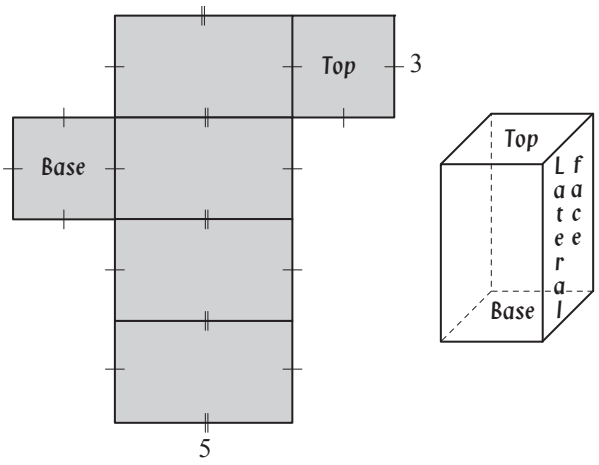
Area of 1 face =

S.A. = = sq. units

continued from page 305

Skill 26.1 Calculating the surface area of rectangular prisms and cubes by using nets (2).

c) Find the surface area of the square prism by finding the area of its net.

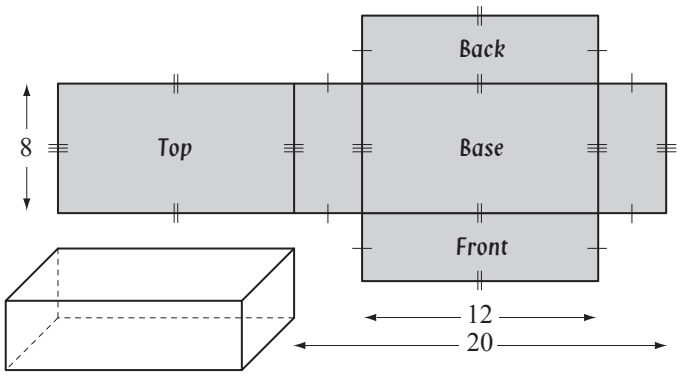


Area: base & top = ...

Area: 4 lateral faces = ...

S.A. = ... = ... sq. units

d) Find the surface area of the rectangular prism by finding the area of its net.



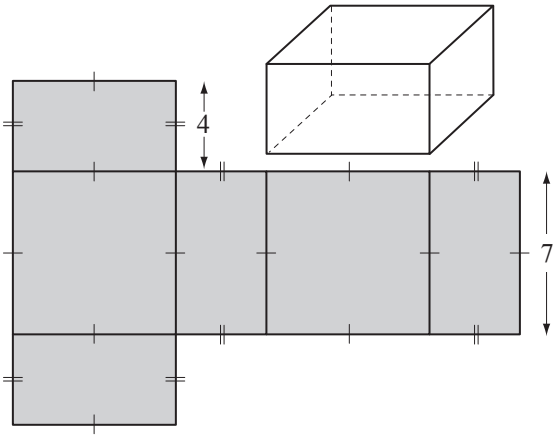
Area: base & top = ...

Area: front & back = ...

Area: 2 other faces = ...

S.A. = ... = ... sq. units

e) Find the surface area of the square prism by finding the area of its net.

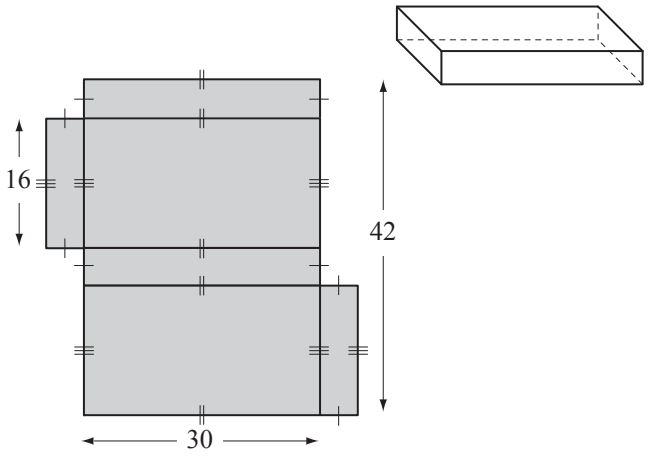


.....

.....

S.A. = ... = ... sq. units

f) Find the surface area of the rectangular prism by finding the area of its net.



.....

.....

.....

S.A. = ... = ... sq. units

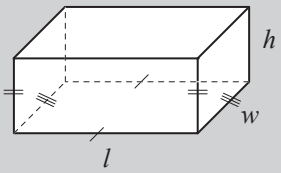
Skill 26.2 Calculating the surface area of rectangular prisms.

- Substitute known values into the formula:

rectangular prism

$S.A. = 2(\text{length} \times \text{width}) + 2(\text{length} \times \text{height}) + 2(\text{width} \times \text{height})$

$S.A. = 2lw + 2lh + 2wh = 2(lw + lh + wh)$



cube

$S.A. = 6(\text{length} \times \text{length}) = 6l^2$

Q. Lewis wants to make a box, with a lid, for his card collection. The box needs a base of 11 cm by 20 cm and must be 12 cm high. How much wood does Lewis need?

A. $S.A. = 2(lw + lh + wh)$ *where* $l = 20$, $w = 11$ *and* $h = 12$

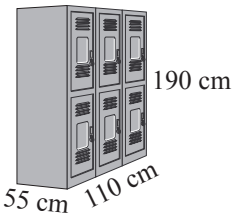
$= 2 \times (20 \times 22 + 20 \times 12 + 11 \times 12)$

$= 2 \times (220 + 240 + 132)$

$= 2 \times 592$

$= \mathbf{1184\ cm^2}$

a) The locker block needs to be resurfaced. What is the surface area of this rectangular prism disregarding its base?



Subtract 1 base area

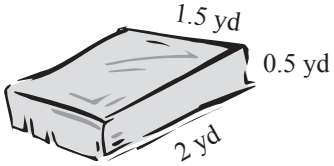
$S.A. = lw + 2lh + 2wh$ *where* $l = 110$, $w = 55$ *and* $h = 190$

$= 110 \times 55 + 2 \times (110 \times 190) + 2 \times (55 \times 190)$

$= 6050 + 2 \times 20{,}900 + 2 \times 10{,}450$

$= 6050 + 41{,}800 + 20{,}900 =$ ____ cm^2

b) Zoe's mattress was torn in removal. What is the minimum amount of mattress ticking needed to re-cover the mattress?



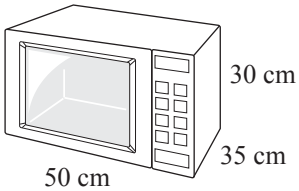
$S.A. = 2(lw + lh + wh)$

$=$ .....

$=$ .....

$=$ ____ yd^2

c) Find the surface area of the microwave.



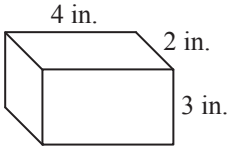
$S.A. =$ .....

$=$ .....

$=$ .....

$=$ ____ cm^2

d) The surface area of the rectangular prism is 52 square inches. What is the *S.A.* if all the dimensions are doubled?



$S.A. =$ .....

$=$ .....

$=$ .....

$=$ ____ in.^2

continues on page 309

Skill 26.3 Calculating the surface area of rectangular composite solids (1).

- Find any unknown side lengths.
- Calculate the area of each face.
- Add together the area of all faces.

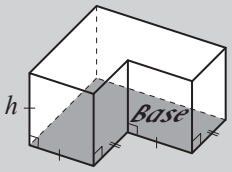
OR

- Identify the base by finding the two, identical parallel faces.
  Hint: A prism does not necessarily sit on its base.
- Substitute values into the formula:

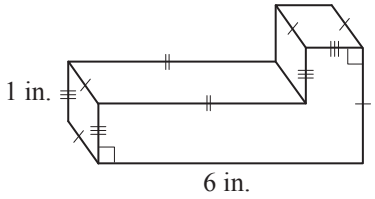
rectangular composite solid

S.A. = Perimeter of base × height + 2 × Area of base

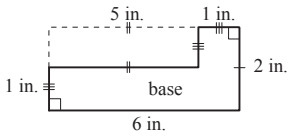
$S.A. = Ph + 2B$



Q. Find the surface area of the prism.



A.

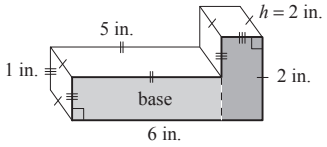


For P, convert to a rectangle

$P = 6 + 1 + 5 + 1 + 1 + 2 = 16$

OR

$P = 6 + 6 + 2 + 2 = 16$

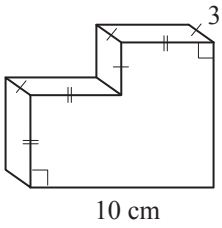


Find unknown side lengths

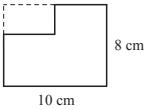
$B = 5 \times 1 + 2 \times 1$
$= 5 + 2 = 7$

$S.A. = Ph + 2B$ where $h = 2$
$= 16 \times 2 + 2 \times 7$
$= 32 + 14 =$ **46 in.²**

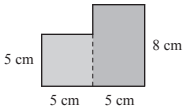
a) Find the surface area of the prism.



For P, convert to a rectangle



For B, find all unknown side lengths



$P = 10 + 10 + 8 + 8 = 36$

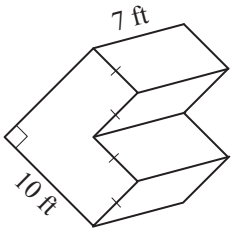
$B = 5 \times 5 + 5 \times 8 = 25 + 40 = 65$

$S.A. = Ph + 2B$ where $h = 3$

$= 36 \times 3 + 2 \times 65$

$= 108 + 130 =$ ____ cm^2

b) Find the surface area of the prism.



$P =$

$B =$

$S.A. = Ph + 2B$

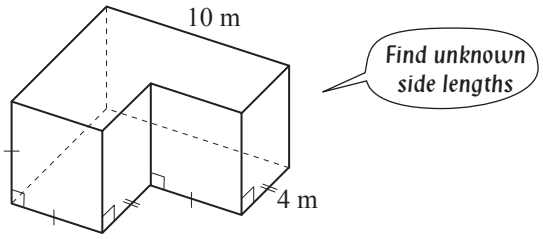
$=$

$=$ ____ ft^2

continued from page 308

Skill 26.3 Calculating the surface area of rectangular composite solids (2).

c) Find the surface area of the prism.



$P =$ .....

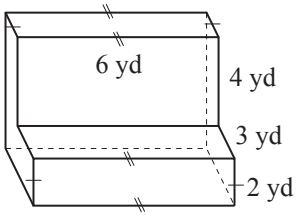
$B =$ .....

$S.A. = Ph + 2B$ where $h =$ .....

$=$ .....

$=$ ..... $=$ m^2

d) Find the surface area of the prism.



$P =$ .....

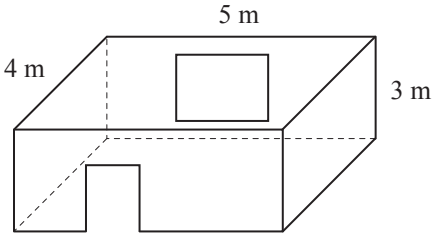
$B =$ .....

$S.A. = Ph + 2B$ .....

$=$ .....

$=$ ..... $=$ yd^2

e) A window 2 m by 1.5 m and a doorway 2 m by 0.8 m are in the plan for this room. Find the area of the walls to be painted.



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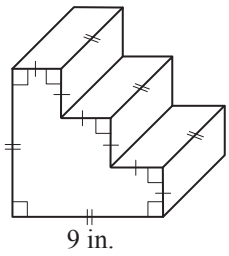
$S.A. =$ ..... $=$ m^2

f) Find the surface area of the prism.



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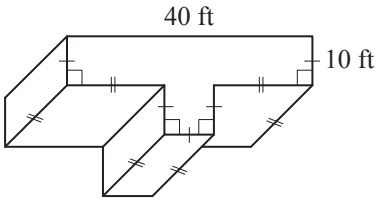
$S.A. =$ ..... $=$ $in.^2$

g) Find the surface area of the prism.



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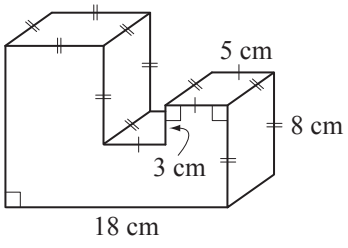
.....

$S.A. =$ ..... $=$ ft^2

h) Find the surface area of the prism.



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.....

.....

$S.A. =$ ..... $=$ cm^2

continues on page 311

Skill 26.4 Calculating the surface area of triangular prisms (1).

- Find any unknown side lengths.
- Calculate the area of each face.
- Add together the area of all faces.

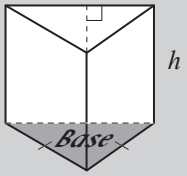
OR

- Substitute values into the formula:

triangular prism

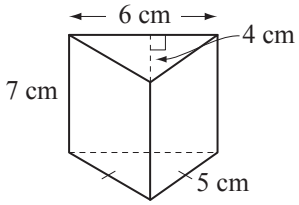
S.A. = Perimeter of base × height + 2 × Area of base

$S.A. = Ph + 2B$

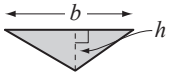


Hint: Do not confuse the height needed to calculate the area of the triangular base, with the height (h) of the prism.

Q. Find the surface area of the triangular prism.



A.



$P = 6 + 5 + 5 = 16$

$B = \frac{1}{2} bh$ where $b = 6, h = 4$

$= \frac{1}{2} \times (6 \times 4)$

$= 12$

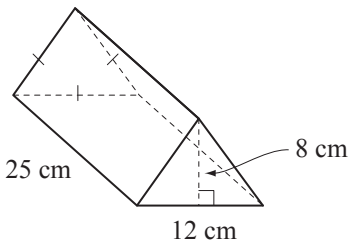
$S.A. = Ph + 2B$ where $h = 7$

$= 16 \times 7 + 2 \times 12$

$= 112 + 24$

$= \mathbf{136\ cm^2}$

a) Find the surface area of the triangular prism.



First find the perimeter of base

$P = 12 + 12 + 12 = 36$

Then find the area of base

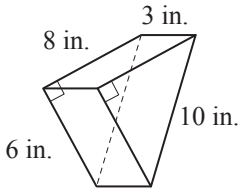
$A = \frac{1}{2} \times (12 \times 8) = 48$

$S.A. = Ph + 2B$ where $h = 25$

$= 36 \times 25 + 2 \times 48$

$= 900 + 96$ = cm²

b) Find the surface area of the triangular prism.



$P =$

$A =$

$S.A. =$

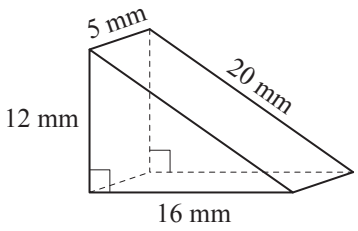
$=$

$=$ in.²

continued from page 310

Skill 26.4 Calculating the surface area of triangular prisms (2).

c) Find the surface area of the triangular prism.



$P =$ .....

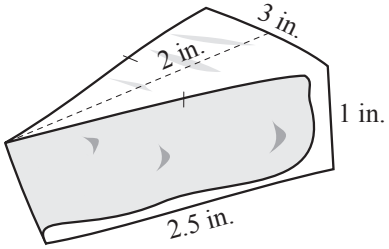
$B =$ .....

$S.A. = Ph + 2B$ where $h =$ .....

$=$ .....

$=$ ..... $=$ mm^2

d) Find the surface area of the triangular prism of cheese.



$P =$ .....

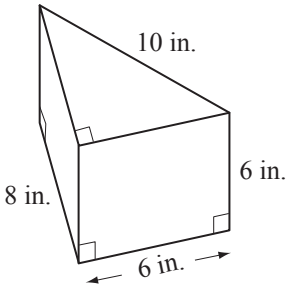
$B =$ .....

$S.A. =$ .....

$=$ .....

$=$ ..... $=$ in.^{2}

e) Find the surface area of the triangular prism.



$P =$ .....

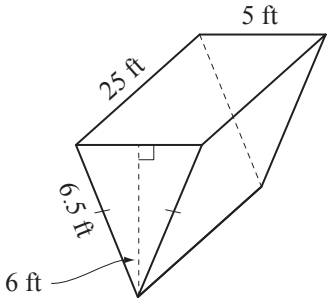
$B =$ .....

$S.A. =$ .....

$=$ .....

$=$ ..... $=$ in.^{2}

f) Find the surface area of the triangular prism.



$P =$ .....

$B =$ .....

$S.A. =$ .....

$=$ .....

$=$ ..... $=$ ft^2

continues on page 313

Skill 26.5 Calculating the surface area of pyramids (1).

- Find any unknown side lengths.
- Calculate the area of each face.
- Add together the area of all faces.

OR

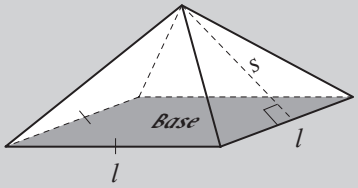
- Substitute values into the formula:

regular square pyramid

$S.A. = \text{Area of base} + 4 \times \text{Area of triangle}$

$S.A. = B + 4 \times \frac{1}{2} ls$

$S.A. = l^2 + 2ls$

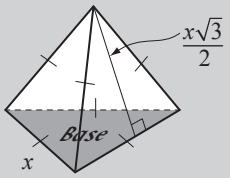


regular triangular pyramid (regular tetrahedron)

$S.A. = 4 \times \text{Area of equilateral triangle}$

$S.A. = 4 \times \frac{1}{2} x \times \frac{x\sqrt{3}}{2}$

$S.A. = x^2\sqrt{3}$

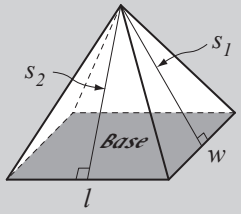


rectangular pyramid

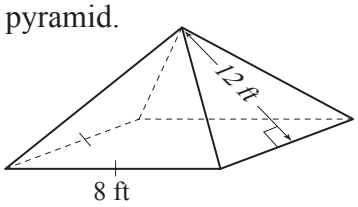
$S.A. = \text{Area of base} + 2 \times \text{Area of triangles left \& right} + 2 \times \text{Area of triangles front \& back}$

$S.A. = B + 2 \times \frac{1}{2} ws_1 + 2 \times \frac{1}{2} ls_2$

$S.A. = lw + ws_1 + ls_2$



Q. Find the surface area of the regular square pyramid.



A. $S.A. = l^2 + 2ls$ *where* $l = 8$ *and* $s = 12$

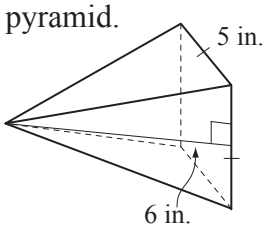
$= 8 \times 8 + 2 \times 8 \times 12$

$= 64 + 16 \times 12$

$= 64 + 192$

$= \mathbf{256\ ft^2}$

a) Find the surface area of the regular square pyramid.



$S.A. = l^2 + 2ls$ *where* $l = 5$ *and* $s = 6$

$= 5 \times 5 + 2 \times 5 \times 6$

$= 25 + 60$ $= \square\ \text{in.}^2$

b) Find the surface area of one of the salt and pepper shakers given that they are regular, square pyramids of base side length 3 cm and slant height 4 cm.

$S.A. = l^2 + 2ls$

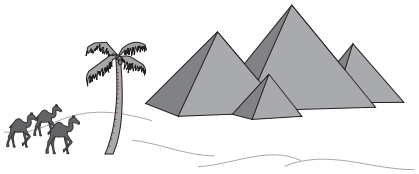
$=$

$=$ $\square\ \text{cm}^2$

continued from page 312

Skill 26.5 Calculating the surface area of pyramids (2).

c) Find the surface area of the largest regular square pyramid, which has a base side length of 200 m and slant height of 250 m.

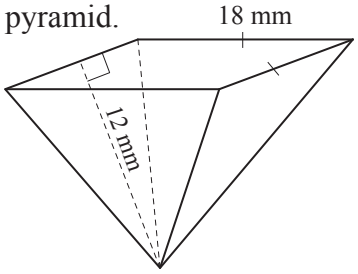


S.A. = ...

.....

..... = $\square$ m^2

d) Find the surface area of the regular square pyramid.

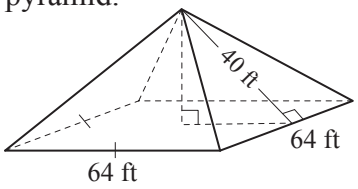


S.A. = ...

.....

..... = $\square$ mm^2

e) Find the surface area of the regular square pyramid.

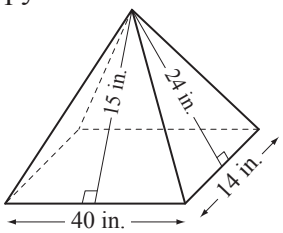


S.A. = ...

.....

..... = $\square$ ft^2

f) Find the surface area of the rectangular pyramid.

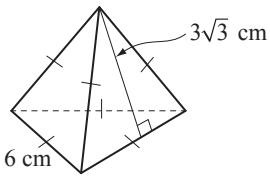


S.A. = ...

.....

..... = $\square$ $in.^2$

g) Find the surface area of the regular tetrahedron. [Give your answer as a radical.]

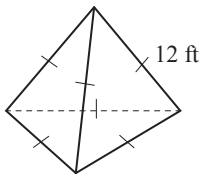


S.A. = ...

.....

..... = $\square$ cm^2

h) Find the surface area of the regular tetrahedron. [Give your answer as a radical.]



S.A. = ...

.....

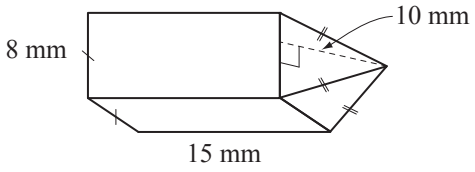
..... = $\square$ ft^2

continues on page 315

Skill 26.6 Calculating the surface area of composite solids (1).

- Break the solid into workable parts.
- Substitute values into the appropriate formula for surface area.
  (see skills 26.2 to 26.5, pages 307 to 312)

Q. Find the total surface area of the obelisk.

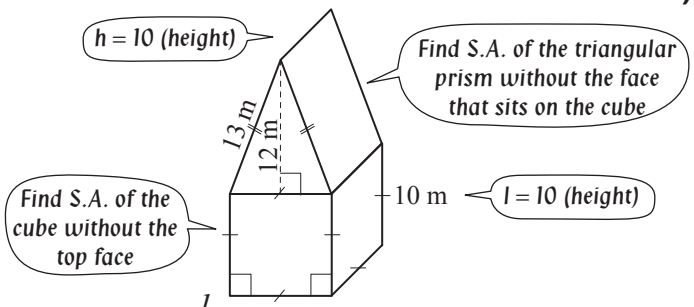


A. *S.A. regular square pyramid (without base)*
$= 2ls$ *where* $l = 8$ *and* $s = 10$
$= 2 \times 8 \times 10$
$= 160$

S.A. square prism (without base)
$= 4lh + l^2$ *where* $l = 8$ *and* $h = 15$
$= 4 \times (8 \times 15) + 8 \times 8$
$= 4 \times 120 + 64 = 544$

S.A. obelisk $= 160 + 544 =$ ***704 mm²***

a) Find the surface area of the solid.



$P = 36$, $B = \frac{1}{2}(10 \times 12) = 60$ and $h = 10$

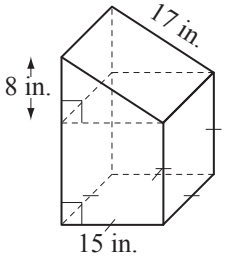
S.A. prism $= Ph + 2B = 36 \times 10 + 2 \times 60 = 480$

S.A. prism − face $= 480 - 100 = 380$

S.A. cube − face $= 5l^2 = 5 \times 100 = 500$

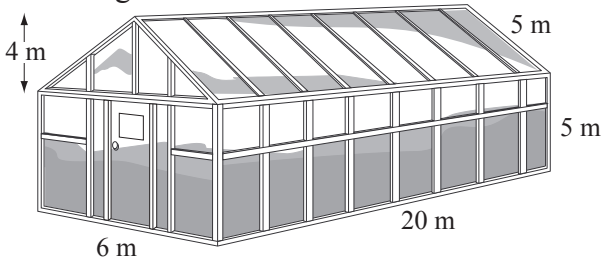
S.A. solid $= 380 + 500$ = ____ m^2

b) Find the surface area of the solid.



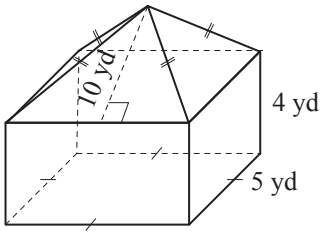
S.A. = ____ = ____ $in.^2$

c) Find the surface area of the glasshouse, excluding its base.



S.A. = ____ = ____ m^2

d) Find the surface area of the obelisk.

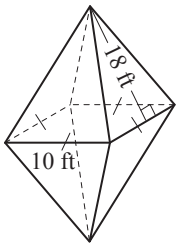


S.A. = ____ = ____ yd^2

continued from page 314

Skill 26.6 Calculating the surface area of composite solids (2).

e) Find the surface area of the octahedron.



S.A. =

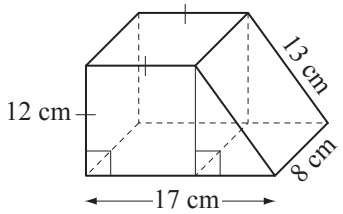
.....

.....

.....

..... = ft^2

f) Find the surface area of the solid.



S.A. =

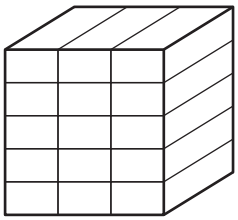
.....

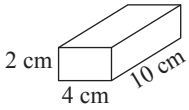
.....

.....

..... = cm^2

g) Lou bought a rectangular box containing 15 tightly packaged erasers. What is the surface area of the box?





S.A. =

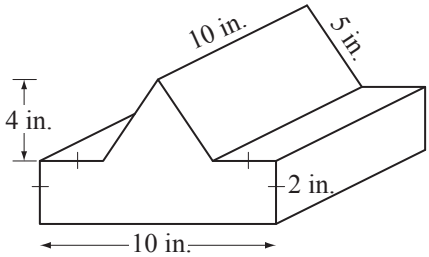
.....

.....

.....

..... = cm^2

h) Find the surface area of the prism.



S.A. =

.....

.....

.....

..... = in.^{2}

continues on page 317

Skill 26.7 Calculating the surface area of basic three-dimensional round solids (1).

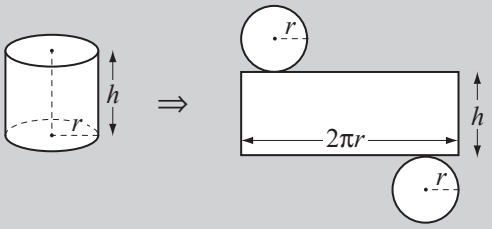
- Substitute values into the formula:

cylinder

$L.A. = 2\pi rh$ $\quad S.A. = 2B + L.A.$

$= 2\pi r^2 + 2\pi rh$

$= 2\pi r\,(r + h)$

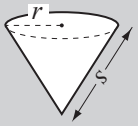


cone

$L.A. = \pi rs$ $\quad S.A. = B + L.A.$

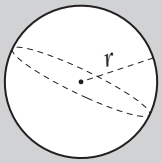
$= \pi r^2 + \pi rs$

$= \pi r\,(r + s)$

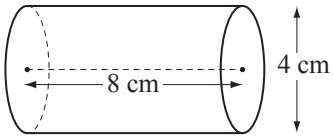


sphere

$S.A. = 4\pi r^2$



Q. Using $S.A. = 2\pi r(r + h)$ and $\pi \approx 3.14$, find the surface area of the cyclinder.



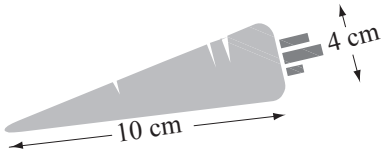
A. $S.A. = 2\pi r\,(r + h)$ *where* $r = 2$ *and* $h = 8$

$= 2 \times 3.14 \times 2 \times (2 + 8)$

$= 12.56 \times 10$

$= \boldsymbol{125.6\ cm^2}$

a) Use $S.A. = \pi r(r + s)$ and $\pi \approx 3.14$ to find the surface area of the conical carrot.

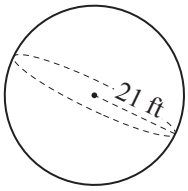


$S.A. = \pi r(r + s)$ *where* $r = 2,\ s = 10$

$\approx 3.14 \times 2 \times (2 + 10)$

$= 6.28 \times 12$ $\quad = \boxed{\text{cm}^2}$

b) Using $S.A. = 4\pi r^2$ and $\pi \approx \frac{22}{7}$, find the surface area of the sphere.



$S.A. =$

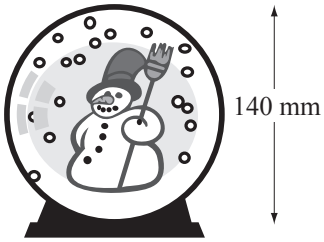
$\approx$

$=$ $\quad = \boxed{\text{ft}^2}$

continued from page 316

Skill 26.7 Calculating the surface area of basic three-dimensional round solids (2).

c) Using $S.A. = 4\pi r^2$ and $\pi \approx \frac{22}{7}$, find the surface area of the snow globe.

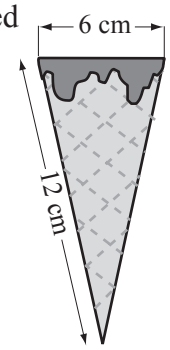


$S.A. =$ .....

$\approx$ .....

$=$ ..... $=$ mm^2

d) Use $S.A. = \pi r(r + s)$ and $\pi \approx 3.14$ to find how much area still needs to be covered in chocolate to cover the whole cone given that 40 cm^2 have been covered so far.

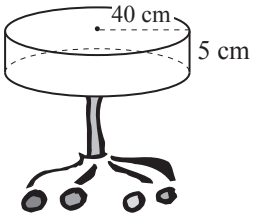


$S.A. =$ .....

$\approx$ .....

$=$ ..... $=$ cm^2

e) Using $S.A. = 2\pi r(r + h)$ and $\pi \approx 3.14$, find the surface area of the cyclindrical stool seat.

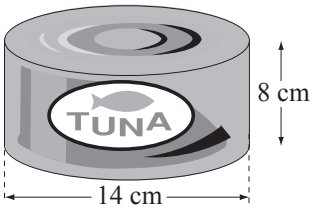


$S.A. =$ .....

$\approx$ .....

$=$ ..... $=$ cm^2

f) Using $S.A. = 2\pi r(r + h)$ and $\pi \approx \frac{22}{7}$, find the surface area of the can of tuna.

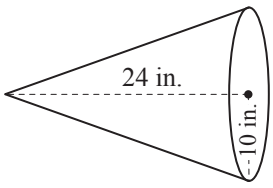


$S.A. =$ .....

$\approx$ .....

$=$ ..... $=$ cm^2

g) Use $\pi \approx 3.14$ to find the surface area of the cone. [Hint: Pythagorean theorem will help.]

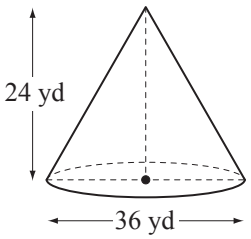


$S.A. =$ .....

$\approx$ .....

$=$ ..... $=$ in.^{2}

h) Use $\pi \approx 3.14$ to find the surface area of the cone. [Hint: Pythagorean theorem will help.]



$S.A. =$ .....

$\approx$ .....

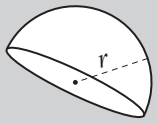
$=$ ..... $=$ yd^2

Skill 26.8 Calculating the surface area of more complex three-dimensional round solids.

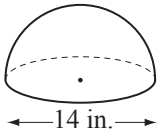
- Substitute values into the appropriate formula:
  (see skills 26.2 to 26.7, pages 307 to 316)
- Adapt the formula where necessary.

hemisphere

$$S.A. = \frac{4\pi r^2}{2} + \pi r^2$$
$$= 3\pi r^2$$



Q. Using $\pi \approx \frac{22}{7}$ find the surface area of the hemisphere.



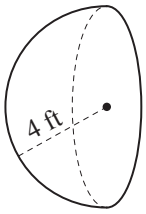
A. *S.A.* $= 3\pi r^2$ *where* $r = 7$

$= 3 \times \frac{22}{1\cancel{4}} \times \cancel{7}^{1} \times 7$

$= 66 \times 7$

$=$ ***462 in.²***

a) Using $\pi \approx 3.14$ find the surface area of the hemisphere.

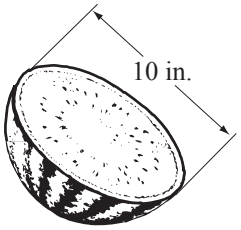


S.A. $= 3\pi r^2$ *where* $r = 4$

$= 3 \times 3.14 \times 4 \times 4$

$= 9.42 \times 16$ $=$ 150.72 ft²

b) Using $\pi \approx 3.14$ find the surface area of the watermelon half.

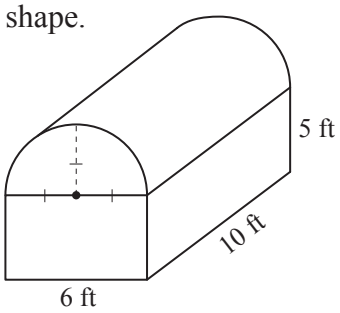


S.A. $=$

$\approx$

$=$ $=$ in.²

c) Use $\pi \approx 3.14$ to find the surface area of the shape.



S.A. prism $=$

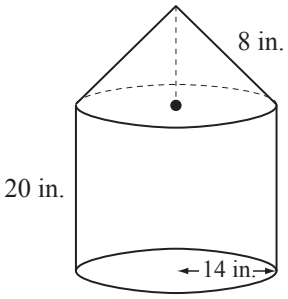
$=$

S.A. cylinder half $=$

$=$

S.A. $=$ $=$ ft²

d) Use $\pi \approx \frac{22}{7}$ to find the surface area of the shape.



L.A. cone $=$

$=$

S.A. cylinder $=$

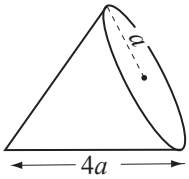
$=$

S.A. $=$ $=$ in.²

Skill 26.9 Expressing the surface area of three-dimensional solids in algebraic form.

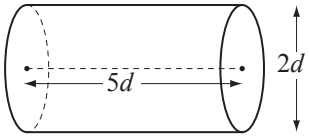
- Substitute values into the appropriate formula for surface area.
  (see skills 26.2 to 26.8, pages 307 to 318)
- Adapt the formula where necessary.

Q. Write a formula for the surface area of the cone.



A. $S.A. = \pi r(r + s)$ *where* $r = a$ *and* $s = 4a$
$= \pi \times a \times (a + 4a)$
$= \pi a \times 5a$
$= \mathbf{5\pi a^2}$

a) Write a formula for the surface area *S.A.* of the cylinder.



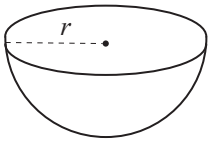
$S.A. = 2\pi r(r + h)$ *where* $r = d$ *and* $h = 5d$

$= 2\pi d(d + 5d)$

$= 2\pi d \times 6d$

$S.A. = 12\pi d^2$

b) Write a formula for the surface area *S.A.* of the hemisphere.



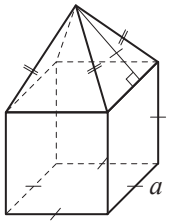
$S.A. =$

$=$

$=$

$S.A. =$

c) Write a formula for the surface area *S.A.* of the obelisk.



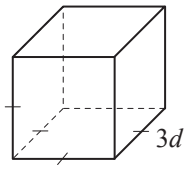
$S.A. =$

$=$

$=$

$S.A. =$

d) Write a formula for the surface area *S.A.* of the cube.



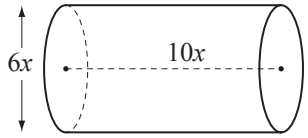
$S.A. =$

$=$

$=$

$S.A. =$

e) Write a formula for the surface area *S.A.* of the cylinder.



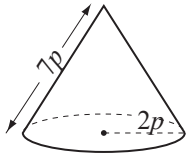
$S.A. =$

$=$

$=$

$S.A. =$

f) Write a formula for the surface area *S.A.* of the cone.



$S.A. =$

$=$

$=$

$S.A. =$