27. [Volume]

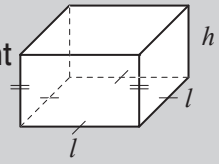
Skill 27.1 Calculating the volume of square and rectangular prisms.

- Substitute known values into the appropriate formula:

square prism

V = length × length × height

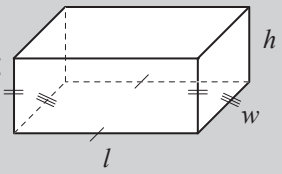
$V = l^2h$



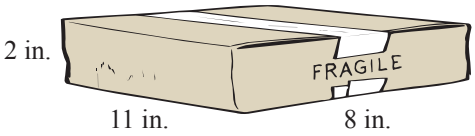
rectangular prism

V = length × width × height

$V = lwh$

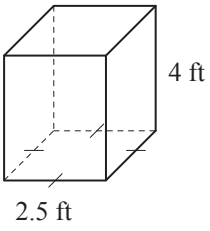


Q. The parcel is a rectangular prism. What is the volume of the parcel?



A. $V = lwh$ where $l = 11$, $w = 8$ and $h = 2$
$= 11 \times 8 \times 2$
$= 88 \times 2$
$= \mathbf{176\ in.^3}$

a) Find the volume of the square prism.

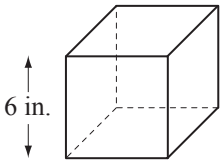


$V = l^2h$ where $l = 2.5$ and $h = 4$

$= 2.5 \times 2.5 \times 4$

$= 2.5 \times 10$ = ft^3

b) Find the volume of the cube.

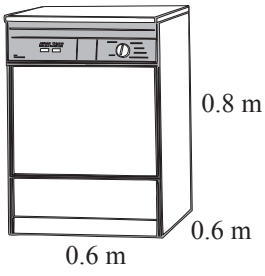


$V = l^3$

=

= = in.^{3}

c) Given that the dishwasher is a square prism, find its volume.

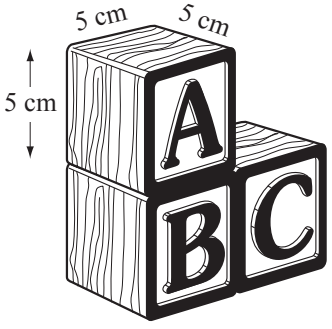


$V =$

=

= = m^3

d) Find the volume of the building blocks stack.



$V =$

=

= = cm^3

continues on page 323

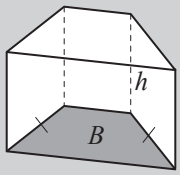
Skill 27.2 Calculating the volume of other prisms (1).

- Find any unknown side lengths.
- Calculate the area of the base. Use known formulae where possible.
  Hint: Do not confuse the height needed to calculate the area of the base, with the height (h) of the prism.
- Substitute known values into the formula:

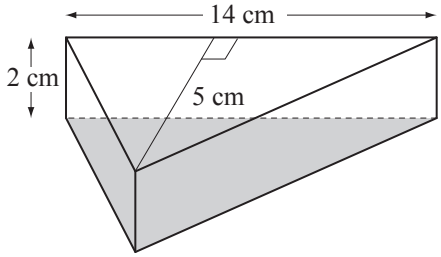
prism

V = Area of base × height of prism

$V = Bh$



Q. Find the volume of the triangular prism using $V = Bh$.



A. $V = Bh$ where $h = 2$ (h = height of prism)

$B = \frac{1}{2}bh$ where $h = 5$ (h = height of triangle)

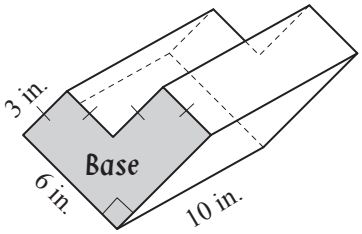
$= \frac{1}{\cancel{2}} \times \cancel{14}^{7} \times 5$ (First calculate area of base)

$= 7 \times 5 = 35$

$V = 35 \times 2$

$= \mathbf{70\ cm^3}$

a) Find the volume of the prism.



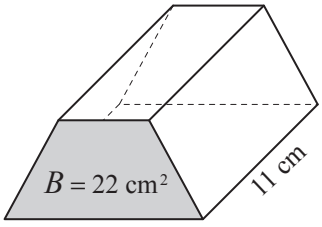
$V = Bh$ where $h = 10$ in.

First calculate area of base

$B = 3 \times 3 + 6 \times 3 = 9 + 18 = 27$

$V = 27 \times 10$ = ____ in.^{3}

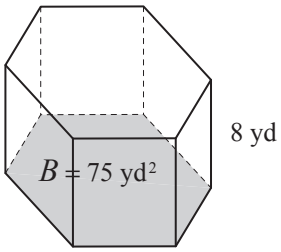
b) Using V = Area of base (B) × height (h), find the volume of the prism.



$V = Bh$

= ____ = ____ cm^3

c) Using $V = Bh$ find the volume of the hexagonal prism.

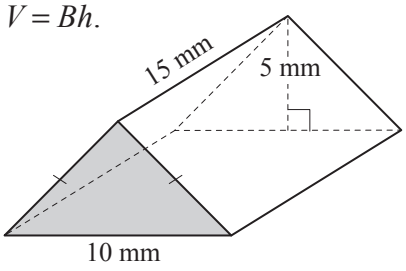


$V = Bh$

$B =$

$V =$ ____ = ____ yd^3

d) Find the volume of the triangular prism using $V = Bh$.



$V = Bh$

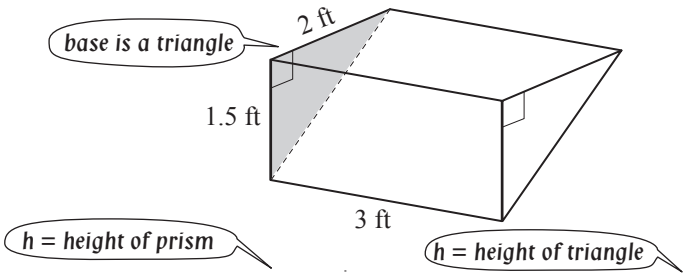
$B =$

$V =$ ____ = ____ mm^3

continued from page 322

Skill 27.2 Calculating the volume of other prisms (2).

e) Find the volume of the prism.

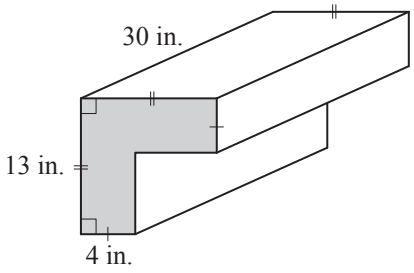


$V = Bh$ where $h = 3$, $B = \frac{1}{2}bh$ where $b = 1.5$ and $h = 2$

$B = \frac{1}{2} \times 1.5 \times 2 = 1.5$ *First calculate area of base*

$V = 1.5 \times 3$ = ____ ft^3

f) Find the volume of the prism.

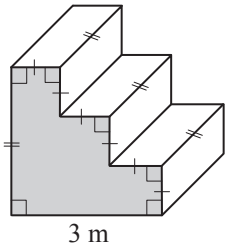


$V = Bh$

$B =$

$V =$ = ____ $in.^3$

g) Find the volume of concrete used to build the steps.

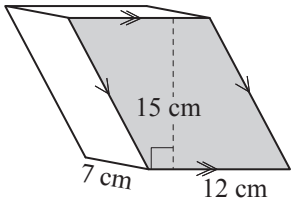


$V =$

$B =$

$V =$ = ____ m^3

h) Find the volume of the prism.

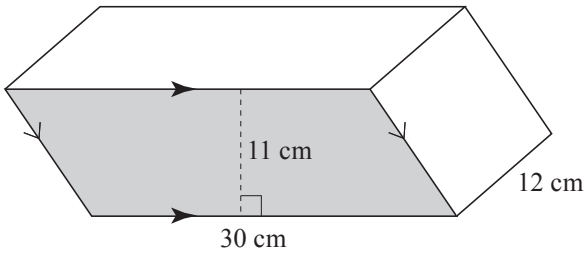


$V =$

$B =$

$V =$ = ____ cm^3

i) Find the volume of the prism.

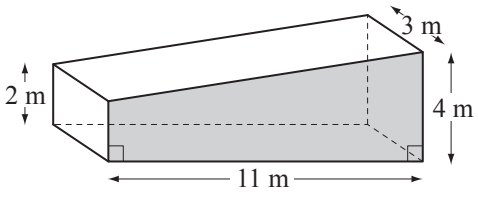


$V =$

$B =$

$V =$ = ____ cm^3

j) Find the volume of the prism.



$V =$

$B =$

$V =$ = ____ m^3

Skill 27.3 Calculating the volume of pyramids.

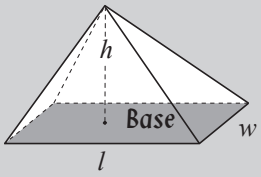
- Substitute known values into the appropriate formulae to find the area of the base.
- Substitute known values into the volume formula.

Hint: Do not confuse the height needed to calculate the area of the base, with the height (h) of the pyramid.

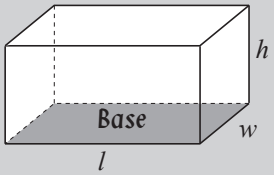
pyramid

$V = \frac{1}{3} \times$ Area of base $\times$ height

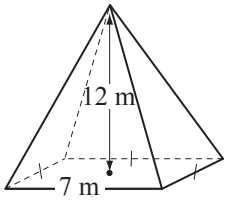
$V = \frac{Bh}{3}$



OR $V = \frac{1}{3} \times$ volume of corresponding prism



Q. Find the volume of the square pyramid.



A. $V = \frac{Bh}{3}$ *where* $h = 12$

$B = l^2$ *where* $l = 7$

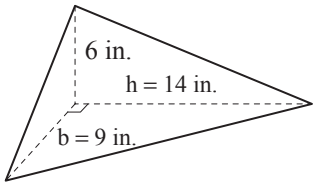
$= 7 \times 7 = 49$

$V = \frac{49 \times 12^{4}}{\cancel{3}}$ Simplify: ÷ 3

$= 49 \times 4$

$= \mathbf{196\ m^3}$

a) Using $V = \frac{Bh}{3}$ find the volume of the triangular pyramid.

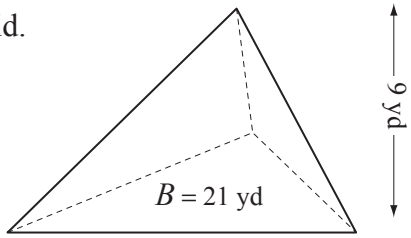


$V = \frac{Bh}{3}$ *where* $h = 6$, $B = \frac{1}{2}bh$ *where* $h = 14$

$B = \frac{1}{\cancel{2}} \times 9 \times \cancel{14}^{7} = 63$

$V = \frac{63 \times \cancel{6}^{2}}{\cancel{3}}$ = in.^{3}

b) Using $V = \frac{Bh}{3}$ find the volume of the triangular pyramid.

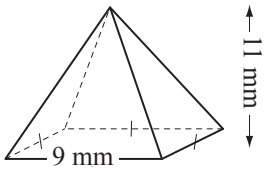


$V = \frac{Bh}{3}$

$B =$

$V =$ = yd^3

c) Find the volume of the square pyramid.

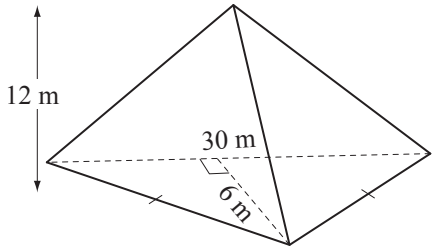


$V =$

$B =$

$V =$ = mm^3

d) Find the volume of the triangular pyramid.



$V =$

$B =$

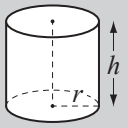
$V =$ = m^3

Skill 27.4 Calculating the volume of basic three-dimensional round solids.

- Substitute known values into the appropriate formula:

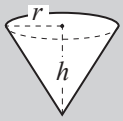
cylinder

$V = \pi r^2 h$



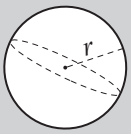
cone

$V = \frac{\pi r^2 h}{3}$

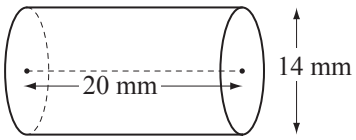


sphere

$V = \frac{4\pi r^3}{3}$



Q. Using $V = \pi r^2 h$ and $\pi \approx \frac{22}{7}$, find the volume of the cylinder.



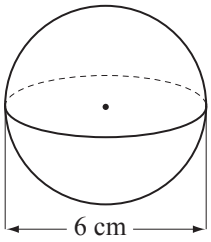
A. $V = \pi r^2 h$ *where* $r = 7$ *and* $h = 20$

$= \frac{22}{\cancel{7}} \times \cancel{7} \times 7 \times 20$ *Simplify: ÷ 7*

$= 154 \times 20$

$= \mathbf{3080\ mm^3}$

a) Using $V = \frac{4\pi r^3}{3}$ and $\pi \approx 3.14$, find the volume of the sphere.

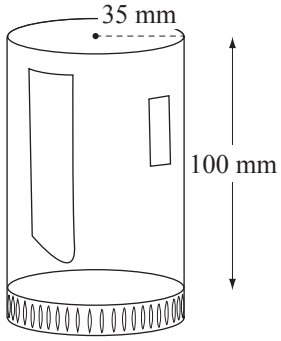


$V = \frac{4\pi r^3}{3}$ *where* $r = 3$ *cm*

$= \frac{4 \times 3.14 \times \cancel{3} \times 3 \times 3}{\cancel{3}}$ *Simplify: ÷ 3*

$= 36 \times 3.14$ = ____ cm^3

b) Using $V = \pi r^2 h$ and $\pi \approx \frac{22}{7}$, find the maximum volume of the glass.

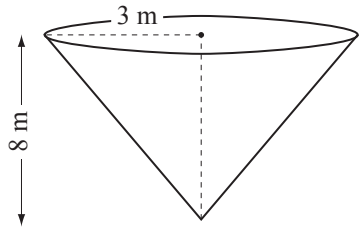


$V =$

$=$

$=$ ____ $= \text{mm}^3$

c) Using $V = \frac{\pi r^2 h}{3}$ and $\pi \approx 3.14$, find the volume of the cone.

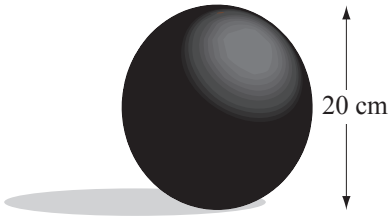


$V =$

$=$

$=$ ____ $= \text{m}^3$

d) Using $V = \frac{4\pi r^3}{3}$ and $\pi \approx 3.14$, find the volume of the sphere.



$V =$

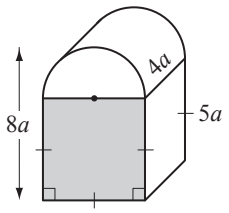
$=$

$=$ ____ $= \text{cm}^3$

Skill 27.5 Expressing the volume of three-dimensional solids in algebraic form.

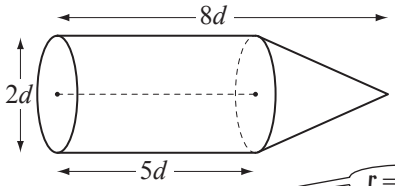
- Substitute values into the appropriate formula for volume. (see skills 27.1 to 27.4, pages 321 to 325)
- Adapt the formula where necessary.

Q. Write a formula for the volume V of the shape.



A.
$V_{sq.\ prism} = l^2h$ *where* $l = 5a$ *and* $h = 4a$
$= 5a \times 5a \times 4a = 100a^3$
$V_{half\ cyl.} = \frac{1}{2}\pi r^2 h$ *where* $r = 3a$ *and* $h = 4a$
$= \frac{1}{2}\pi \cdot 9a^2 \cdot 4a = 18\pi a^3$
$V_{shape} = 100a^3 + 18\pi a^3 = \mathbf{2a^3(50 + 9\pi)}$

a) Write a formula for the volume V of the shape.



r = d and h = 5d

$V_{cyl} = \pi r^2 h = \pi \cdot d^2 \cdot 5d = 5\pi d^3$

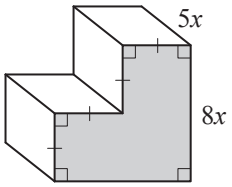
r = d and h = 3d

$V_{cone} = \frac{\pi r^2 h}{3} = \frac{1}{3} \cdot \pi \cdot d^2 \cdot 3d = \pi d^3$

$V_{shape} = 5\pi d^3 + \pi d^3$

$V =$

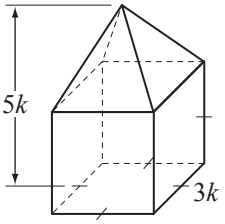
b) Write a formula for the volume V of the prism.



$V =$

$V =$

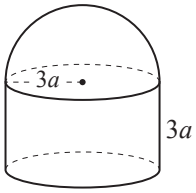
c) Write a formula for the volume V of the shape.



$V =$

$V =$

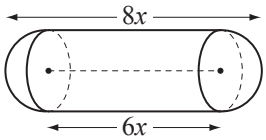
d) Write a formula for the volume V of the shape.



$V =$

$V =$

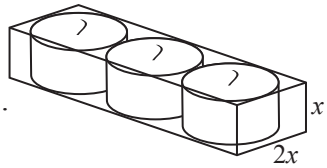
e) Write a formula for the volume V of the capsule.



$V =$

$V =$

f) A rectangular box contains 3 identical cylindrical candles placed with no room to move. Write a formula for the volume of the box which is not occupied by the candles.



$V =$

$V =$

Skill 27.6 Calculating volume in relation to capacity.

- Substitute known values into the appropriate formula.
- Use the conversion factors between cubic units and capacity units:

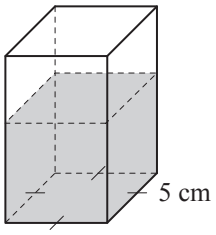
$1000\ cm^3 = 1000\ mL = 1\ L$

$1000\ L = 1\ m^3$

Q. A rectangular swimming pool is 20 m long and 12 m wide. If its average depth is 2 m, how many liters of water would you need to fill the pool? [Hint: $1000\ L = 1\ m^3$]

A. $V = lwh$ *where* $l = 20$, $w = 12$ *and* $h = 2$
$= 20 \times 12 \times 2$
$= 20 \times 24$
$= 480\ m^3$ (Convert m^3 to L)
$= \mathbf{480{,}000\ L}$

a) The vase has 0.5 liters of water in it. Find the depth of the water. [Hint: $1000\ cm^3 = 1\ L$]



$V = l^2h$ *where* $l = 5$ *and* $h =$ *unknown depth*

(Using 0.5 L = 500 mL)

$500 = 5 \times 5 \times h \Rightarrow 25h = 500$

$25h \div 25 = 500 \div 25$

$h = 20$ cm

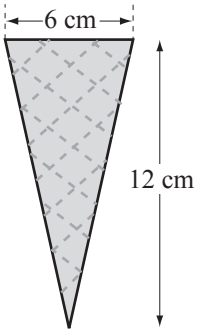
b) A rectangular fish tank, with dimensions 20 cm by 15 cm by 10 cm, is half full of water. How many milliliters of water would you need to fill the fish tank? [Hint: $1\ mL = 1\ cm^3$]

=

=

=

c) Using $V = \frac{\pi r^2 h}{3}$ and $\pi \approx 3.14$, find how much ice cream could fit exactly inside this cone. [Hint: $1\ mL = 1\ cm^3$]

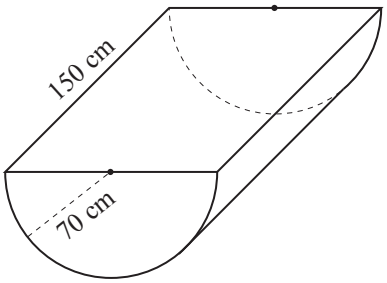


$V =$

=

= = mL

d) Using $\pi \approx \frac{22}{7}$ find the maximum volume of water the trough could hold. [Hint: $1000\ cm^3 = 1\ L$]



$V =$

=

= = L

Skill 27.7 Calculating volume in relation to length and area.

- Substitute known values into the appropriate formulae for area and volume.

Q. A rectangular prism with volume 216 cm^3 has a height of 6 cm and a width of 5 cm. Calculate the length of the prism.

A. *V = lwh where V = 216, w = 5 and h = 6*
216 = l · 5 · 6
30 · l = 216
l = 216 ÷ 30 (Divide 21.6 by 3)
l = 7.2 cm

a) If a cube has a surface area of 54 cm^2, what is the volume of the cube?

S.A. = 6l^2 and V = l^3 (In a cube: l = w = h)

54 = 6l^2 so l^2 = $\frac{54}{6}$ = 9 and l = 3

V = 3^3 = ____ cm^3

b) A rectangular prism with volume 189 in.^{3} has a height of 3 in. and a length of 7 in. Calculate the width of the prism.

____ in.

c) If a cube has a surface area of 96 yd^2, what is the volume of the cube?

____ yd^3

d) If a cube has a surface area of 150 ft^2, what is the volume of the cube?

____ ft^3

e) A rectangular long jump pit holds 13.5 m^3 of sand. If the pit is 9 m long and 3 m wide, how deep is the sand?

____ m

f) How many metal cubes of side length 4 mm need to be melted down to produce a single cube of side length 8 mm?

____

g) A rectangular fish tank can hold 30,000 cm^3 when full. If the tank is 20 cm wide and 30 cm long, how deep is the water?

____ cm

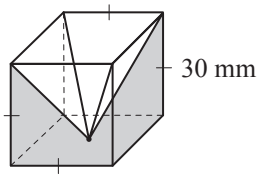
h) How many metal cubes of side length 3 inches need to be melted down to produce a single cube of side length 9 inches?

____

Skill 27.8 Calculating the volume of composite solids.

- Substitute values into the appropriate formula for volume.

Q. What is the volume of the shaded space, given a square pyramid of the cube length is carved from the cube?



A. V_1 *cube* $= l^3$

V_2 *square pyramid* $= \frac{Bh}{3}$ *where* $B = l^2$ *and* $h = l$

$= \frac{l^3}{3}$

$V_1 - V_2 = l^3 - \frac{l^3}{3}$

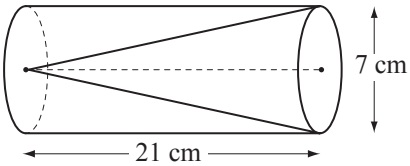
$V = \frac{2l^3}{3}$ *where* $l = 30$

$V = \frac{2 \times \overset{10}{\cancel{30}} \times 30 \times 30}{\cancel{3}_1}$ Simplify: ÷ 3

$V = 20 \times 900$

$= \mathbf{18{,}000\ mm^3}$

a) How much less is the volume of the cone than the volume of the cylinder of the same height? (Use $\pi \approx \frac{22}{7}$)



V_1 *of a cylinder* $= \pi r^2 h$, V_2 *of a cone* $= \frac{\pi r^2 h}{3}$

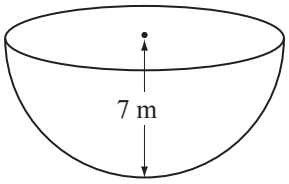
$V_1 - V_2 = \pi r^2 h - \frac{\pi r^2 h}{3}$

$= \frac{2\pi r^2 h}{3}$ *where* $r = \frac{7}{2}$ *and* $h = 21$

$= \cancel{2} \times \frac{\cancel{22}^{11}}{\cancel{7}} \times \frac{\cancel{7}}{\cancel{2}} \times \frac{7}{\cancel{2}} \times \cancel{21}^{7} \times \frac{1}{\cancel{3}}$ Simplify

$= 11 \times 7 \times 7$ = cm³

b) Using $\pi \approx \frac{22}{7}$ find the volume of the hemisphere.



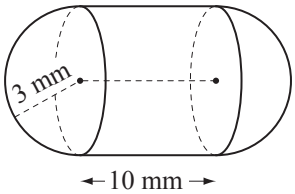
$V =$

$=$

$=$

$=$ = m³

c) Using $\pi \approx 3.14$ find the volume of the shape.

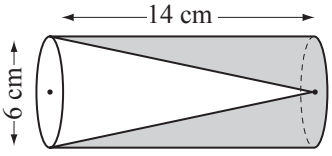


$V =$

$=$

$=$ = mm³

d) Using $\pi \approx \frac{22}{7}$ find the volume of the shaded space.



$V =$

$=$

$=$ = cm³