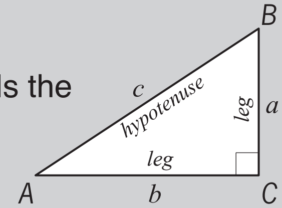
28. [Pythagorean Theorem / Trigonometry]

Skill 28.1 Recognizing Pythagorean theorem.

MMMauve 1 1 2 2 3 3 4 4
MMLime 1 1 2 2 3 3 4 4

Pythagorean Theorem: $a^2 + b^2 = c^2$

For any right triangle, the square of the length of the hypotenuse (c) equals the sum of the squares of the lengths of the legs (a and b).

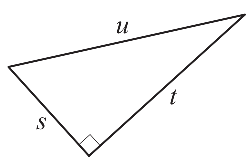


- Determine which is the longest side of the right triangle (hypotenuse).
  Hints: In a triangle, the vertices are labeled with capital letters.
  Any side length of a triangle is usually labeled with a lower case letter (the same as the letter labeling the opposite vertex or angle).
- Identify correct relations of Pythagorean theorem or those derived from it:
  - The square of the length of a leg equals the difference between the square of the length of the hypotenuse and the square of the length of the other leg.

  $a^2 = c^2 - b^2$ and $b^2 = c^2 - a^2$

Q. Which statements of Pythagorean theorem are correct?

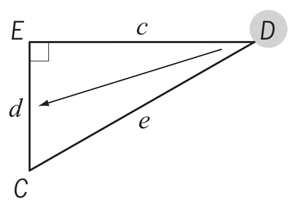
A) $t^2 + u^2 = s^2$
B) $u^2 = s^2 + t^2$
C) $s^2 = u^2 - t^2$



A. *Pythagorean statements are:* $u^2 = s^2 + t^2$
or $s^2 + t^2 = u^2$
or $s^2 = u^2 - t^2$
or $t^2 = u^2 - s^2$

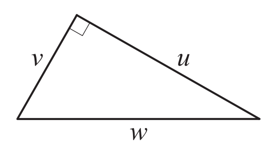
The correct statements are **B** *and* **C**.

a) Which letter marks the leg opposite to angle D in this right triangle?



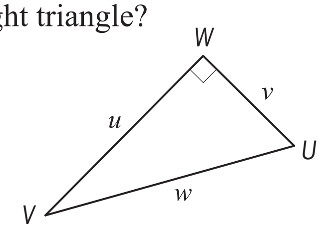
b) Which statements of Pythagorean theorem are correct?

A) $w^2 = u^2 + v^2$
B) $u^2 = v^2 + w^2$
C) $v^2 = w^2 - u^2$



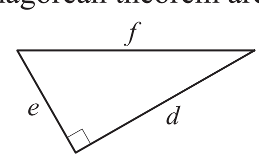
and

c) Which letter marks the leg adjacent to angle V in this right triangle?



d) Which statements of Pythagorean theorem are correct?

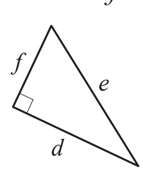
A) $d^2 = e^2 + f^2$
B) $f^2 = e^2 + d^2$
C) $e^2 = f^2 - d^2$

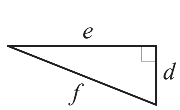


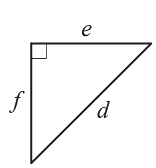
and

e) Connect the following Pythagorean relationships to their corresponding diagram:

$e^2 = d^2 - f^2$ $f^2 = d^2 + e^2$ $d^2 = e^2 - f^2$

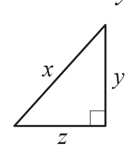


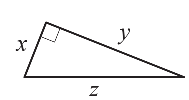


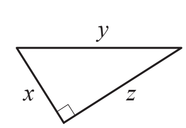


f) Connect the following Pythagorean relationships to their corresponding diagram:

$x^2 = z^2 - y^2$ $y^2 = x^2 + z^2$ $z^2 = x^2 - y^2$







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Skill 28.2 Solving quadratic equations.

- Calculate the square numbers on both sides of the equation.
- Isolate the variable on the left-hand side of the equation.
- Evaluate and simplify the right-hand side of the equation.
- Take the square root of both sides of the equation to find the value of the unknown.

Q. Find the positive solution for b: $12^2 + b^2 = 15^2$

A.
$12^2 + b^2 = 15^2$
$144 + b^2 = 225$
$144 - 144 + b^2 = 225 - 144$
$b^2 = 81$
$\sqrt{b^2} = \sqrt{81}$ ($81 = 9 \times 9$)
$b = \mathbf{9}$

a) Find the positive solution for c: $12^2 + 16^2 = c^2$

$144 + 256 = c^2$

$c^2 = 400$

$\sqrt{c^2} = \sqrt{400}$

$c =$ 20

b) Find the positive solution for a: $a^2 + 15^2 = 17^2$

$a^2 +$

$a^2 =$

$a =$

$a =$ ☐

c) Find the positive solution for b: $5^2 + b^2 = 13^2$

$25 + b^2 =$

$b^2 =$

$b =$

$b =$ ☐

d) Find the positive solution for a: $a^2 + 20^2 = 25^2$

$a^2 =$

$a =$

$a =$ ☐

e) Find the positive solution for b: $24^2 + b^2 = 25^2$

$b =$

$b =$ ☐

f) Find the positive solution for c: $9^2 + 12^2 = c^2$

$c =$

$c =$ ☐

g) Find the positive solution for c: $10^2 + 24^2 = c^2$

$c =$ ☐

h) Find the positive solution for b: $40^2 + b^2 = 50^2$

$b =$ ☐

i) Find the positive solution for c: $7^2 + 24^2 = c^2$

☐

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Skill 28.3 Finding the length of the hypotenuse when the lengths of the legs of a right triangle are given.

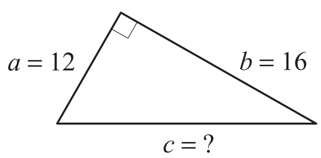
MMMauve 1 1 2 2 3 3 4 4
MMLime 1 1 2 2 3 3 4 4

Pythagorean Theorem: $a^2 + b^2 = c^2$

- Identify the given side lengths on the diagram.
- State Pythagorean theorem.
- Substitute the values into Pythagorean theorem.
- Isolate the unknown quantity on the left-hand side of the equation.
- Evaluate and simplify the right-hand side of the equation.
- Take the square root of both sides of the equation to find the value of the unknown.

Hint: The most common triplets of numbers that make Pythagorean theorem true (Pythagorean triples) are: (3, 4, 5) (5, 12, 13) (8, 15, 17) (7, 24, 25) e.g. $5^2 + 12^2 = 13^2$

Q. Using Pythagorean theorem $a^2 + b^2 = c^2$, find the length of the hypotenuse of this triangle.



A. *a = 12 and b = 16*

$a^2 + b^2 = c^2$

$12^2 + 16^2 = c^2$

$c^2 = 12^2 + 16^2$

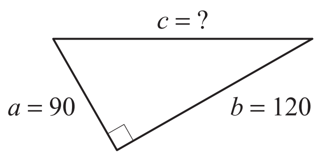
$c^2 = 144 + 256$

$c^2 = 400$

$\sqrt{c^2} = \sqrt{400}$

$c = \mathbf{20}$

a) Using Pythagorean theorem, find the length of the hypotenuse of this triangle.



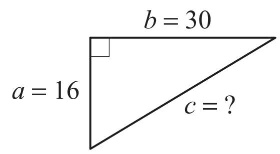
$90^2 + 120^2 = c^2$

$c^2 = 8100 + 14{,}400$

$c^2 = 22{,}500$

$\sqrt{c^2} = \sqrt{22{,}500}$ c = ______

b) Find the length of the hypotenuse of this triangle.



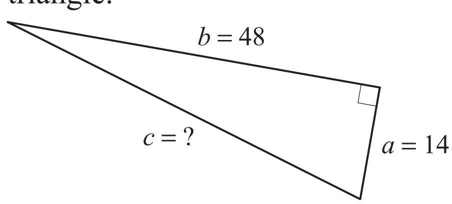
....

$c^2 =$

$c^2 =$

$c =$ c = ______

c) Find the length of the hypotenuse of this triangle.



....

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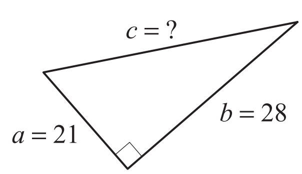
$c =$ c = ______

d) Find the length of the hypotenuse of this triangle.



....

....

....

c = ______

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Skill 28.4 Finding the length of a leg when the lengths of the other leg and the hypotenuse of a right triangle are given.

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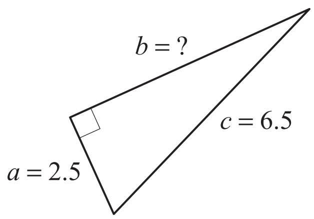
MMLime 1 1 2 2 3 3 4 4

Pythagorean Theorem: $a^2 + b^2 = c^2$

- Identify the given side lengths on the diagram.
- State Pythagorean theorem.
- Substitute the values into Pythagorean theorem.
- Isolate the unknown quantity on the left-hand side of the equation.
- Evaluate and simplify the right-hand side of the equation.
- Take the square root of both sides of the equation to find the value of the unknown.

Hint: The most common triplets of numbers that make Pythagorean theorem true (Pythagorean triples) are: (3, 4, 5) (5, 12, 13) (8, 15, 17) (7, 24, 25). e.g. $3^2 + 4^2 = 5^2$

Q. Using Pythagorean theorem, find the length of the side labeled *b*.



A. $a = 2.5$ and $c = 6.5$

$a^2 + b^2 = c^2$

$2.5^2 + b^2 = 6.5^2$

$b^2 = 6.5^2 - 2.5^2$

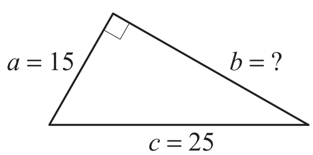
$b^2 = 42.25 - 6.25$

$b^2 = 36$

$\sqrt{b^2} = \sqrt{36}$

$b = \mathbf{6}$

a) Using Pythagorean theorem, find the length of the side labeled *b*.



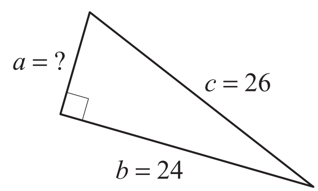
$15^2 + b^2 = 25^2$

$b^2 = 625 - 225$

$b^2 = 400$

$\sqrt{b^2} = \sqrt{400}$ $b =$ ☐

b) Find the length of the side labeled *a*.



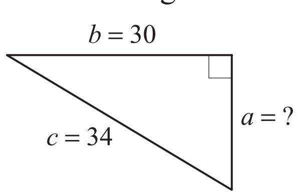
....

$a^2 =$

$a^2 =$

$a =$ $a =$ ☐

c) Find the length of the side labeled *a*.



....

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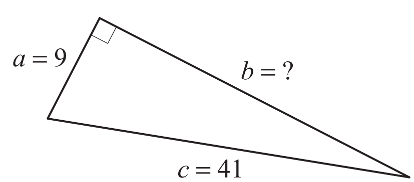
$a =$ $a =$ ☐

d) Find the length of the side labeled *b*.



....

....

....

$b =$ $b =$ ☐

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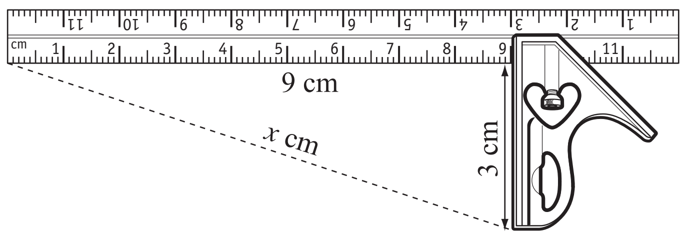
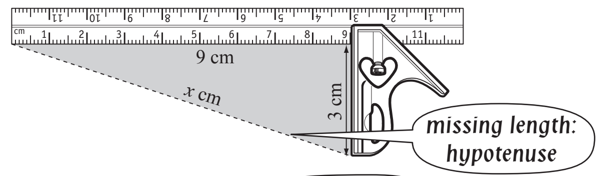
Skill 28.5 Applying Pythagorean theorem (1).

- Locate or draw a right triangle in the diagram.
- Identify the given side lengths in this right triangle.
- Identify the unknown side length in this right triangle and label it with a letter.
- Use Pythagorean theorem to find the unknown side length.
  (see skill 28.3, page 333 and skill 28.4, page 334)
- Simplify the radical. (see skill 11.11, page 133)

Q. Find the missing length in this diagram showing a T-square. [Reduce the radical to simplest form.]



A.



$x^2 = 3^2 + 9^2$ (Pythagorean theorem)

$x^2 = 9 + 81$

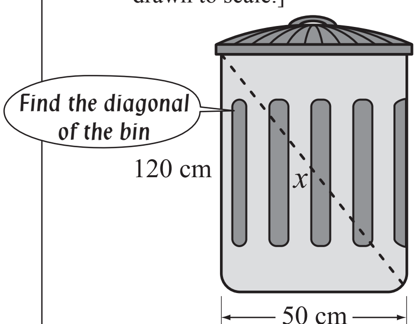
$x^2 = 90$

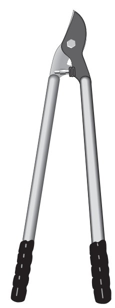
$x = \sqrt{9 \times 10}$

$x = \sqrt{9} \times \sqrt{10}$

$x = \mathbf{3\sqrt{10}}$

a) Would clipping shears, 125 cm long, fit inside this garbage bin with its lid on? [Objects not drawn to scale.]





Pythagorean theorem

$x^2 = 50^2 + 120^2$

$x^2 = 2500 + 14,400$

$x^2 = 16,900$

$x = \sqrt{16,900}$

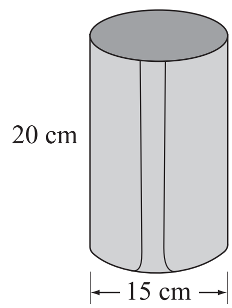
$x = 130$

clipper = 125 cm

125 cm < 130 cm

clipper fits inside the bin **yes**

b) Would a 26 cm long paint brush fit inside this tin with its lid on? [Objects not drawn to scale.]



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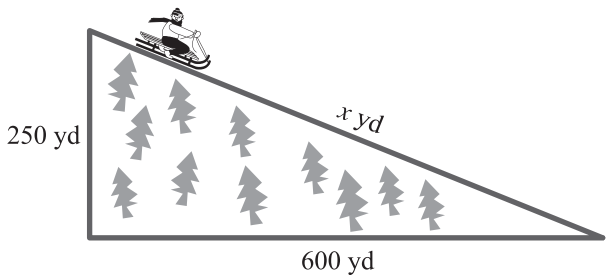
Skill 28.5 Applying Pythagorean theorem (2).

c) How far down this mountain slope is the sleigh descending?



$x^2 = 250^2 + 600^2$

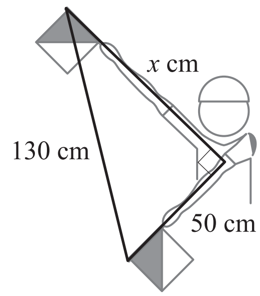
$x^2 =$

$x^2 =$

$x =$

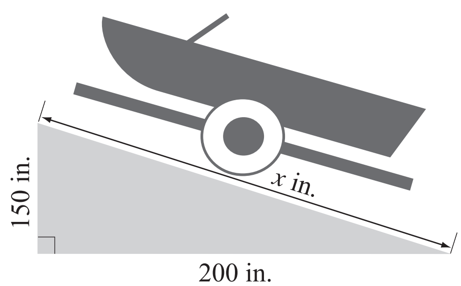
$x =$ .... yd

d) What is the distance marked x on this diagram showing the semaphore which signals letter I?



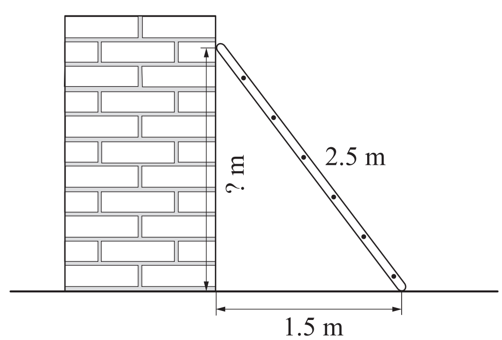
$x =$ .... cm

e) How long is the ramp on which the boat descends?



$x =$ .... in.

f) A 2.5 m long ladder is leaning against a wall and its end is 1.5 m from the base of the wall. How high up the wall is the ladder reaching?



$x =$ .... m

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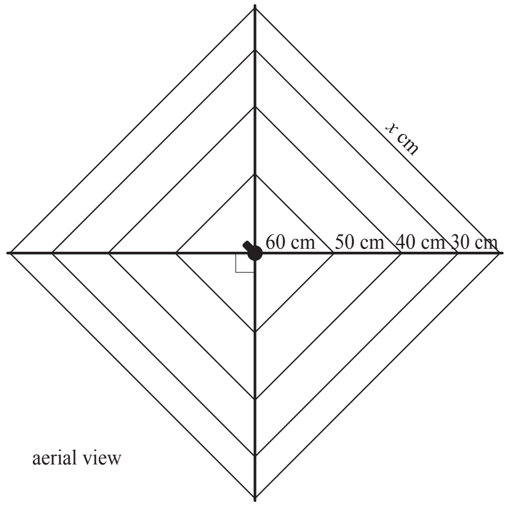
Skill 28.6 Finding a side length in isosceles right triangles (1).

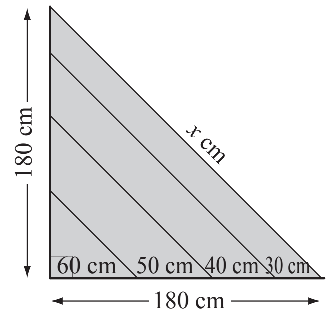
MMMauve 1 1 2 2 3 3 4 4
MMLime 1 1 2 2 3 3 4 4

- Use Pythagorean theorem in the isosceles right triangle to find an unknown side length. (see skill 28.3, page 333 and skill 28.4, page 334)
- Simplify the radical. (see skill 11.11, page 133)

Q. How much wire was used for the outside square of this clothes line?
[Reduce the radical to simplest form.]



A.



$x^2 = 180^2 + 180^2$ (Pythagorean theorem)

$x^2 = 2 \times 180^2$

$x = \sqrt{2 \times 180^2}$

$x = 180\sqrt{2}$

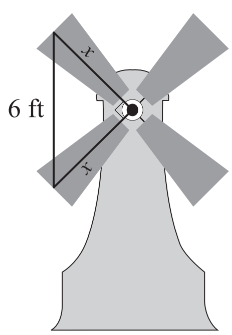
$\text{Perimeter wire} = 4x$

$= 4 \times 180\sqrt{2}$

$= \mathbf{720\sqrt{2}\ cm}$

(approx. 1000 cm)

a) How long is each blade of this windmill, if they are all the same length and the distance between the tips of two consecutive blades is 6 m? [Reduce the radical to simplest form.]



$x^2 + x^2 = 6^2$ (Pythagorean theorem)

$2x^2 = 36$

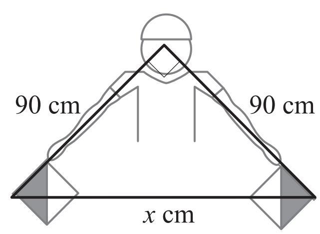
$2x^2 \div 2 = 36 \div 2$

$x^2 = 18$

$x = \sqrt{18}$

$x = \sqrt{2 \times 9}$ = $3\sqrt{2}$ ft

b) What is the distance between the flags when this semaphore is signaling letter N as shown in the diagram below? [Reduce the radical to simplest form.]



$x^2 = 90^2 + 90^2$

$x^2 = 2 \times$

$x =$

$x =$ ______ cm

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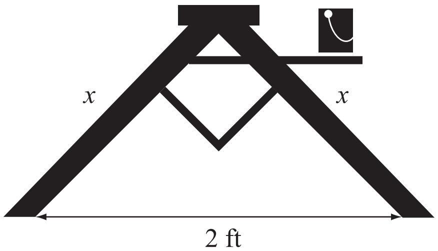
Skill 28.6 Finding a side length in isosceles right triangles (2).

c) How long are this ladder's legs, if they are 2 m apart? [Reduce the radical to simplest form.]

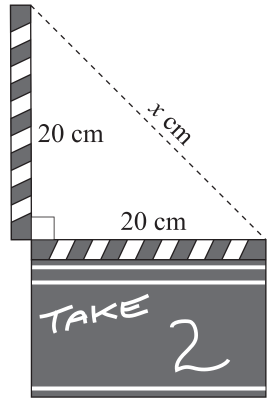


$x^2 + x^2 = 2^2$

$x =$

$x =$ ft

d) Find the missing length in this diagram showing a clapboard. [Reduce the radical to simplest form.]



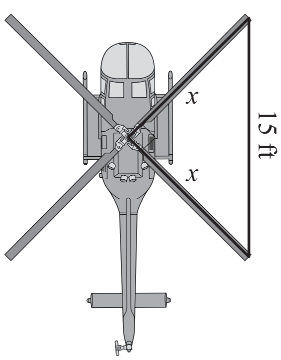
$x^2 =$

$x^2 =$

$x =$

$x =$ cm

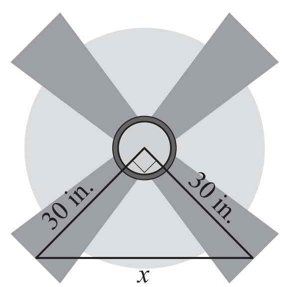
e) How long is each of these helicopter blades, if they are all the same length and the distance between the tips of two consecutive blades is 15 ft? [Reduce the radical to simplest form.]



x^2

ft

f) Find the missing length in this diagram showing a ceiling fan. [Reduce the radical to simplest form.]



in.

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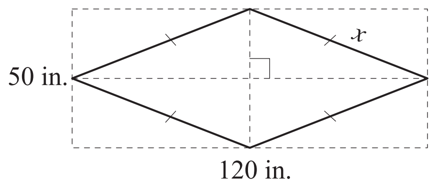
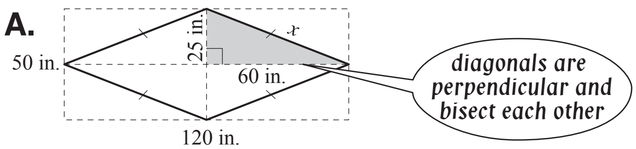
Skill 28.7 Applying Pythagorean theorem to find the perimeter of two-dimensional shapes.

- Highlight a right triangle in the diagram.
- Identify the given side lengths in this right triangle.
- Identify the missing side length in this right triangle and label it with a letter.
- Use Pythagorean theorem to find the missing side length.
  (see skill 28.3, page 333 and skill 28.4, page 334)
- Calculate the perimeter of the two-dimensional shape. (see skill 24.1, pages 281)

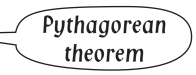
Q. Find the perimeter of this rhombus by first calculating the missing side length.



A.



$x^2 = 25^2 + 60^2$



$x^2 = 625 + 3600$

$x^2 = 4225$

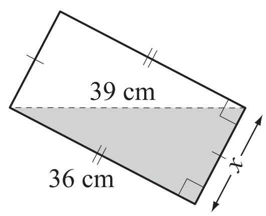
$x = \sqrt{25 \times 169}$

$x = 5 \times 13$

$x = 65$

$P = 4 \times 65$ in. = **260 in.**

a) Find the perimeter of this rectangle by first calculating the missing side length.



$x^2 + 36^2 = 39^2$

$x^2 = 1521 - 1296$

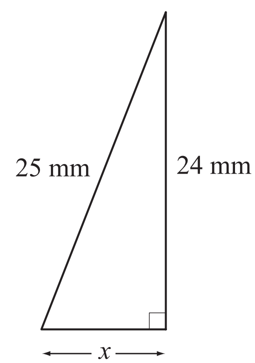
$x^2 = 225$

$x = \sqrt{225}$

$x = 15$

$P = 15 + 15 + 36 + 36$ = ______ cm

b) Find the perimeter of this triangle by first calculating the missing side length.



$x^2 + 24^2 = 25^2$

$x^2 =$

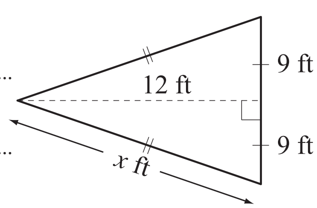
$x^2 =$

$x =$

$x =$

$P = 15 + 15 + 36 + 36$ = ______ mm

c) Find the perimeter of this isosceles triangle by first calculating the missing side length.



....

$x^2 =$

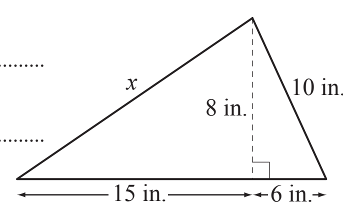
$x^2 =$

$x =$

$x =$

$P =$ = ______ ft

d) Find the perimeter of this triangle by first calculating the missing side length.



....

....

....

....

....

$P =$ = ______ in.

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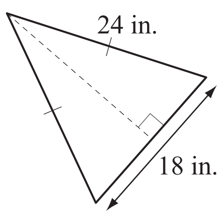
Skill 28.8 Applying Pythagorean theorem to find the area of two-dimensional shapes.

- Highlight a right triangle in the diagram.
- Identify the given side lengths in this right triangle.
- Identify the missing side length in this right triangle and label it with a letter.
- Use Pythagorean theorem to find the missing side length.
  (see skill 28.3, page 333 and skill 28.4, page 334)
- Calculate the area of the two-dimensional shape. (see skills 25.1 to 25.5, pages 291 to 295)

Q. Find the area of this triangle.
[Reduce the radical to simplest form.]



A. *Let x = perpendicular height*

$x^2 = 24^2 - 9^2$

$x^2 = 576 - 81$

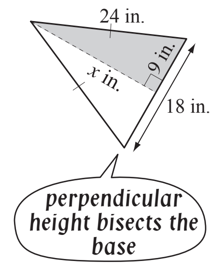
$x^2 = 495$

$x = \sqrt{495}$ (495 = 9 × 55)

$x = 9\sqrt{55}$

Area of triangle $= \frac{1}{2} bh$

$= \frac{1}{2} \times 18 \times 9\sqrt{55} = \mathbf{81\sqrt{55}\ in.^2}$



a) Find the area of this parallelogram by first calculating the missing side length.

$x^2 + 12^2 = 15^2$

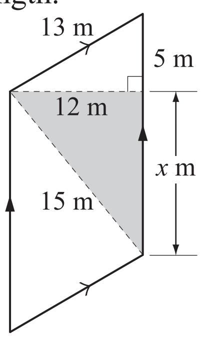
$x^2 = 225 - 144$

$x^2 = 81$

$x = \sqrt{81} \Rightarrow x = 9$

$base_{parall.} = 5 + 9 = 14$

$A_{parall.} = bh = 14 \times 12 =$ ______ m^2



b) Find the area of this right triangle by first calculating the missing side length.

$x^2 + 40^2 = 41^2$

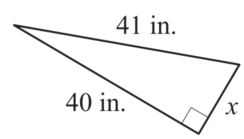
$x^2 =$ ....

$x^2 =$ ....

$x =$ ....

$A_{triangle} =$ ....

$=$ .... $=$ ______ $in.^2$



c) Find the area of this square by first calculating the missing side length.

$x^2 + x^2 = 12^2$

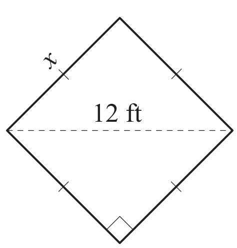
$2x^2 =$ ....

$x^2 =$ ....

$x =$ ....

$A_{square} =$ ....

$=$ .... $=$ ______ ft^2



d) Find the area of this rectangle by first calculating the missing side length.

....

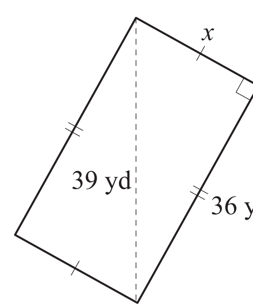
....

....

....

$A_{rectangle} =$ ....

$=$ .... $=$ ______ yd^2



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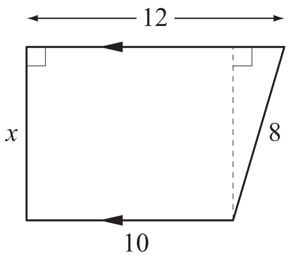
Skill 28.9 Applying Pythagorean theorem in a variety of two-dimensional diagrams.

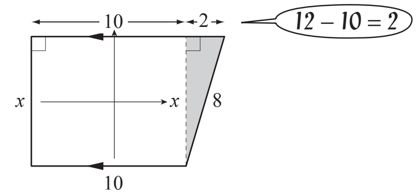
MMMauve 1 1 2 2 3 3 4 4
MMLime 1 1 2 2 3 3 4 4

- Highlight a right triangle in the diagram.
- Identify the given lengths in this right triangle.
- Identify the missing length in this right triangle.
- Use Pythagorean theorem to find the missing length. (see skill 28.3, page 333 and skill 28.4, page 334)

Q. Find the missing length in this trapezoid.
[Reduce the radical to simplest form.]



A.



$x^2 + 2^2 = 8^2$ (Pythagorean theorem)

$x^2 = 64 - 4$

$x^2 = 60$

$x = \sqrt{60}$

$x = \sqrt{4 \times 15}$

$x = \mathbf{2\sqrt{15}}$

a) Find the missing length in this rectangle.

$x^2 + 40^2 = 85^2$ (Pythagorean theorem)

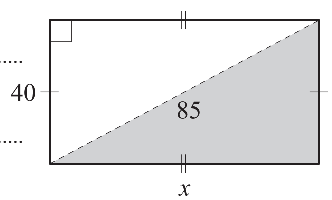
$x^2 = 7225 - 1600$

$x^2 = 5625$

$x = \sqrt{5625}$

$x = \sqrt{25 \times 225}$

$x = 5 \times 15$ = ______



b) Find the missing length in this triangle.

$x^2 =$ ....

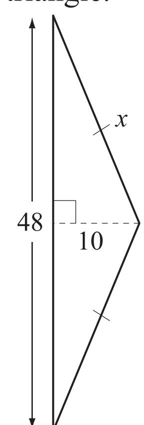
$x^2 =$ ....

$x^2 =$ ....

$x =$ ....

$x =$ ....

$x =$ .... = ______



c) Find the missing length in this triangle.
[Give your answer as a radical.]

....

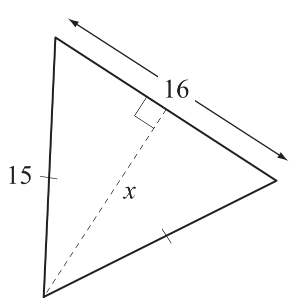
$x^2 =$ ....

$x^2 =$ ....

$x =$ ....

$x =$ ....

$x =$ .... = ______



d) Find the missing length in this trapezoid.
[Reduce the radical to simplest form.]

....

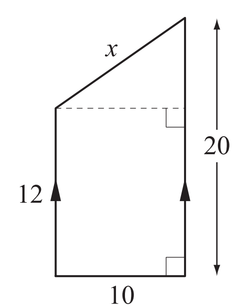
$x^2 =$ ....

$x^2 =$ ....

$x =$ ....

$x =$ ....

$x =$ .... = ______



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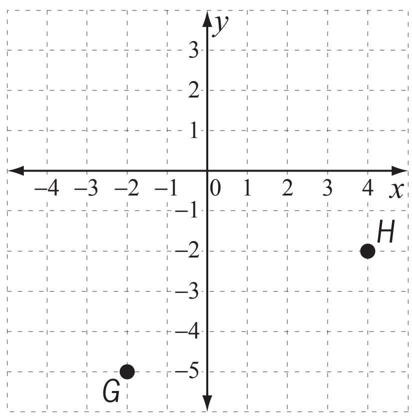
continues on page 343

Skill 28.10 Applying Pythagorean theorem to find the distance between two points located on a coordinate plane (1).

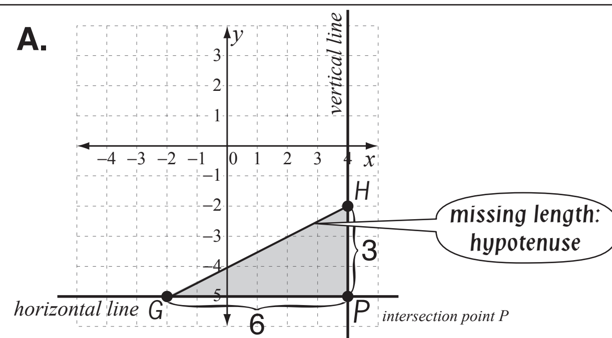
MMMauve 1 1 2 2 3 3 4 4
MMLime 1 1 2 2 3 3 4 4

- Draw a horizontal line through one of the points.
- Draw a vertical line through the other point.
- Mark the point at the intersection of these lines.
- Join the three points (the two given points and the point at the intersection) to form a triangle.
- Count the units along the horizontal and vertical sides of the triangle.
- Use Pythagorean theorem in this right triangle to find the hypotenuse.
  (see skill 28.3, page 333)
- Simplify the radical. (see skill 11.11, page 133)

Q. Find the distance GH in this coordinate plane.
[Reduce the radical to simplest form.]

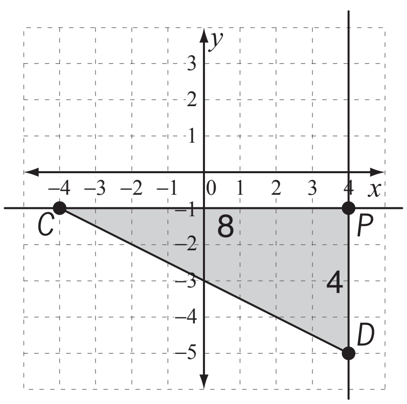


A.



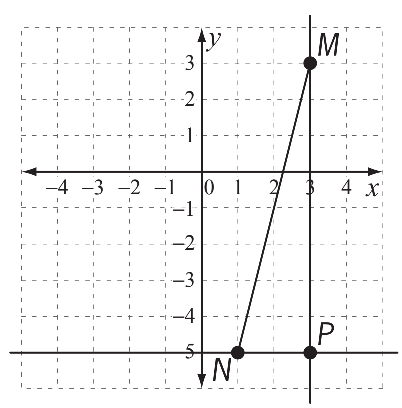
$GH^2 = GP^2 + PH^2$ (Pythagorean theorem)
$GH^2 = 6^2 + 3^2$
$GH^2 = 36 + 9$
$GH^2 = 45$
$GH = \sqrt{45}$
$GH = \sqrt{9 \times 5}$
$GH = \mathbf{3\sqrt{5}}$

a) Find the distance CD in this coordinate plane.
[Reduce the radical to simplest form.]



$CD^2 = CP^2 + PD^2$
$CD^2 = 8^2 + 4^2$
$CD^2 = 80$
$CD = \sqrt{80}$
$CD = \sqrt{16 \times 5}$ = ☐

b) Find the distance MN in this coordinate plane.
[Reduce the radical to simplest form.]



$MN^2 = MP^2 + PN^2$
....
....
....
= ☐

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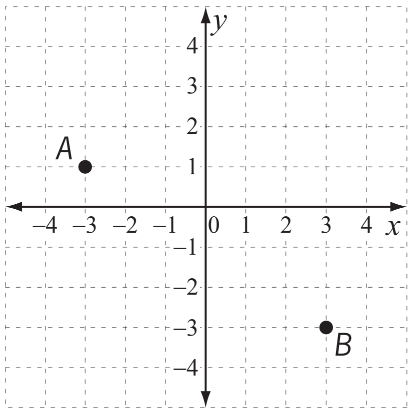
continued from page 342

Skill 28.10 Applying Pythagorean theorem to find the distance between two points located on a coordinate plane (2).

MMMauve 1 1 2 2 3 3 4 4
MMLime 1 1 2 2 3 3 4 4

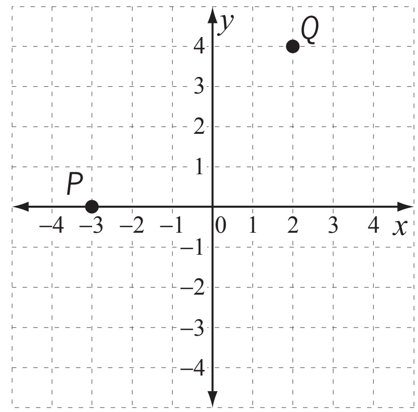
c) Find the distance AB in this coordinate plane.
[Reduce the radical to simplest form.]



$AB^2 =$

=

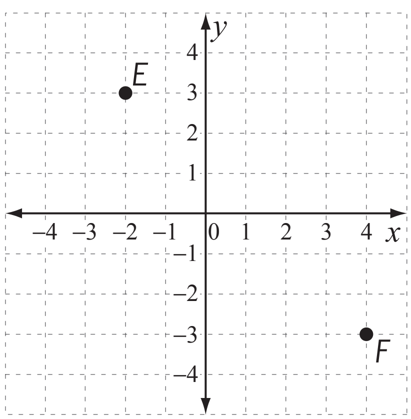
d) Find the distance PQ in this coordinate plane.
[Give your answer as a radical.]



$PQ^2 =$

=

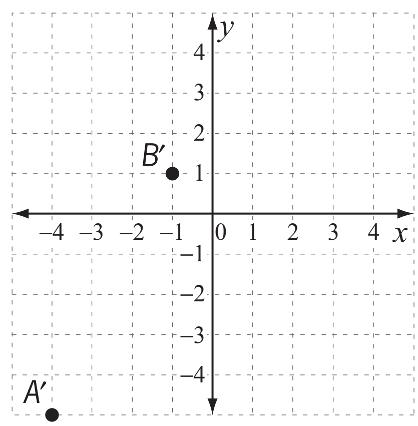
e) Find the distance EF in this coordinate plane.
[Reduce the radical to simplest form.]



$EF^2 =$

=

f) Find the distance $A'B'$ in this coordinate plane.
[Reduce the radical to simplest form.]



$A'B'^2 =$

=

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Skill 28.11 Recognizing trigonometric functions (sine, cosine, tangent).

Trigonometric ratio (function) sine

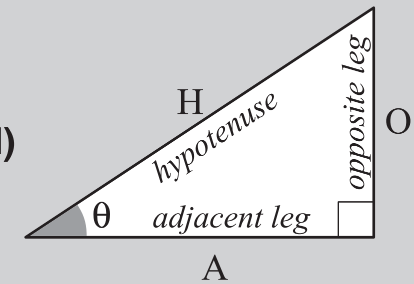
$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$ $\sin\theta = \dfrac{O}{H}$ **S**ine**O**pposite**H**ypotenuse **(SOH)**

Trigonometric ratio (function) cosine

$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$ $\cos\theta = \dfrac{A}{H}$ **C**osine**A**djacent**H**ypotenuse **(CAH)**

Trigonometric ratio (function) tangent

$\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$ $\tan\theta = \dfrac{O}{A}$ **T**angent**O**pposite**A**djacent **(TOA)**

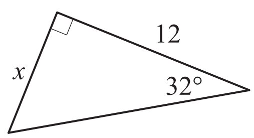


- Identify the hypotenuse of the triangle, and the opposite and adjacent sides corresponding to the marked angle (α - alpha, β - beta, θ - theta).
- Label each side of the triangle with H, O and A.
- Decide which two sides of the triangle are given OR which side and angle of the triangle are given.
- Use one of the SOH - CAH - TOA relations to decide which trigonometric ratio can be used to find the unknown angle OR the unknown side.

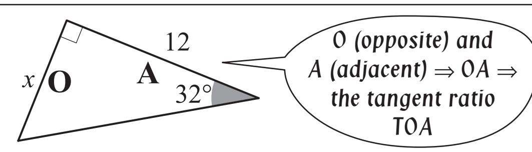
  Hint: Use the SOH - CAH - TOA rules to remember the trigonometric ratios.

Q. Which trigonometric ratio would be used to find the unknown side x?

A) sine
B) cosine
C) tangent



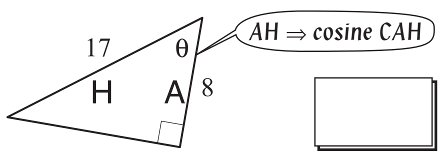
A.



The answer is ***C***.

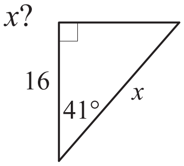
a) Which trigonometric ratio would be used to find angle θ?

A) sine
B) cosine
C) tangent



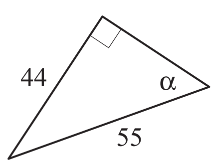
b) Which trigonometric ratio would be used to find the unknown side x?

A) sine
B) cosine
C) tangent



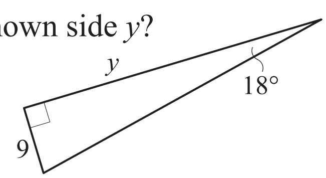
c) Which trigonometric ratio would be used to find angle α?

A) sine
B) cosine
C) tangent



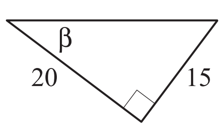
d) Which trigonometric ratio would be used to find the unknown side y?

A) sine
B) cosine
C) tangent



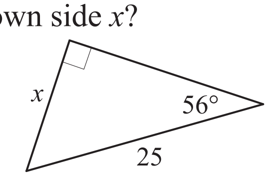
e) Which trigonometric ratio would be used to find angle β?

A) sine
B) cosine
C) tangent



f) Which trigonometric ratio would be used to find the unknown side x?

A) sine
B) cosine
C) tangent



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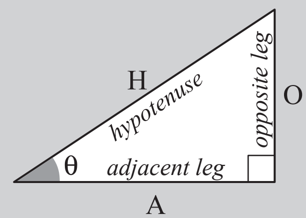
Skill 28.12 Calculating the value of basic trigonometric ratios in right triangles.

MMMauve 1 1 2 2 3 3 4 4
MMLime 1 1 2 2 3 3 4 4

$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$ $\sin\theta = \dfrac{O}{H}$ **S**ine**O**pposite**H**ypotenuse **(SOH)**

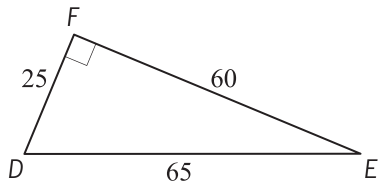
$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$ $\cos\theta = \dfrac{A}{H}$ **C**osine**A**djacent**H**ypotenuse **(CAH)**

$\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$ $\tan\theta = \dfrac{O}{A}$ **T**angent**O**pposite**A**djacent **(TOA)**

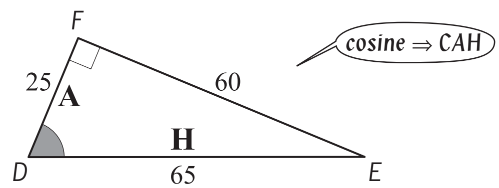


- Mark the angle whose trigonometric ratio is required.
- Label each side of the triangle with H, O and A.
- Use one of the SOH - CAH - TOA relations to calculate the required trigonometric ratio.

Q. Calculate the value of cos *D* in this triangle.



A.

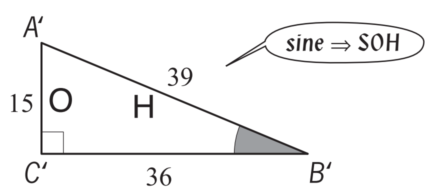


$\cos D = \dfrac{\text{adjacent}}{\text{hypotenuse}}$

$= \dfrac{25}{65} \begin{matrix}\div 5\\ \div 5\end{matrix}$

$= \mathbf{\dfrac{5}{13}}$

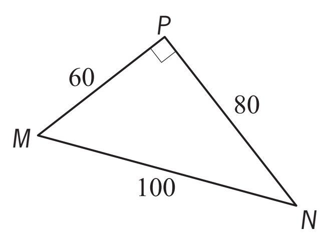
a) Calculate the value of sin *B′* in this triangle.



$\sin B' = \dfrac{O}{H}$

$= \dfrac{15}{39} \begin{matrix}\div 3\\ \div 3\end{matrix}$ = ☐

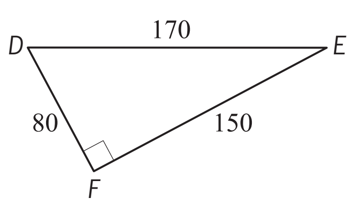
b) Calculate the value of tan *M* in this triangle.



$\tan M =$ ....

= .... = ☐

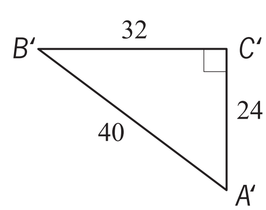
c) Calculate the value of cos *E* in this triangle.



$\cos E =$ ....

= .... = ☐

d) Calculate the value of sin *A′* in this triangle.



$\sin A' =$ ....

= .... = ☐

continues on page 347

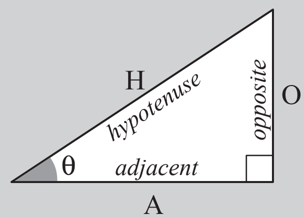
Skill 28.13 Finding an unknown side of a right triangle when a trigonometric ratio of an angle and another side of the triangle are given (1).

MMMauve 1 1 2 2 3 3 4 4
MMLime 1 1 2 2 3 3 4 4

$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$ $\sin\theta = \dfrac{O}{H}$ **S**ine**O**pposite**H**ypotenuse **(SOH)**

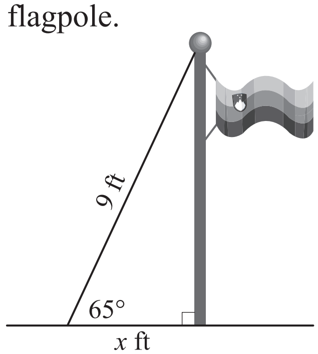
$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$ $\cos\theta = \dfrac{A}{H}$ **C**osine**A**djacent**H**ypotenuse **(CAH)**

$\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$ $\tan\theta = \dfrac{O}{A}$ **T**angent**O**pposite**A**djacent **(TOA)**

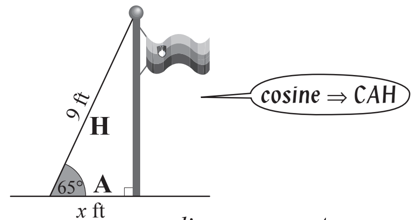


- Label each side of the triangle with H, O and A.
- Use the SOH or CAH or TOA relation corresponding to the given trigonometric ratio value.
- Substitute the numeric values in the relation.
- Solve the equation for the unknown side length.

Q. A 9 ft support wire is attached to a flagpole and makes an angle of 65° with the ground. If cos 65° ≈ 0.42, find the approximate distance from the end of the wire to the base of the flagpole.



A.



$\cos 65° = \dfrac{adjacent}{hypotenuse} = \dfrac{A}{H}$

$\dfrac{42}{100} \times \dfrac{x}{9}$ (Cross multiply)

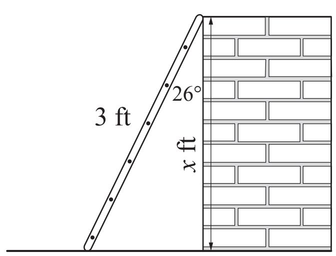
$42 \cdot 9 = 100 \cdot x$

$100x = 378$

$x = 378 \div 100$

$\boldsymbol{x = 3.78\ ft}$

a) A 3 foot ladder is leaning against a wall and makes an angle of 26° with the wall. If cos 26° ≈ 0.89, how high up the wall is the top of the ladder?



$\cos 26° = \dfrac{adjacent}{hypotenuse}$

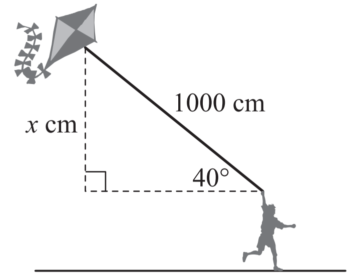
$\dfrac{89}{100} = \dfrac{x}{3}$ (Cross multiply)

$100x = 267$

$x = 267 \div 100$

$x =$ .... ft

b) A kite's string makes an angle of 40° with the horizon. The string length is 1000 cm and sin 40°≈ 0.64. If the boy's height is 160 cm, how high above the ground is the kite flying?



$\sin 40° =$ ....

$x =$ .... $height =$ .... cm

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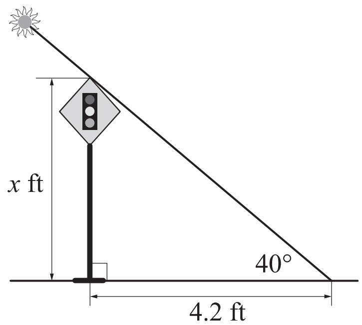
Skill 28.13 Finding an unknown side of a right triangle when a trigonometric ratio of an angle and another side of the triangle are given (2).

c) A road sign casts a shadow which is 4.2 feet long when the sun is at an angle of 40° in the sky. If tan 40° = 0.84, find the height of the road sign.
[Give your answer correct to 2 decimal places.]



tan 40° =

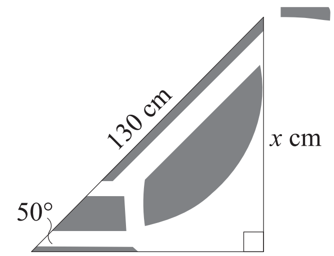
....

....

....

$x =$ .... ft

d) In this profile view the vacuum cleaner makes an angle of 50° with the ground. If sin 50°≈ 0.77, how high above the ground is the handle of the vacuum cleaner?



sin 50° =

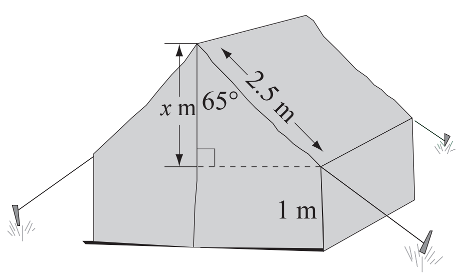
....

....

....

$x =$ .... cm

e) If cos 65° ≈ 0.42, what is the height of this tent above the ground?



cos 65° =

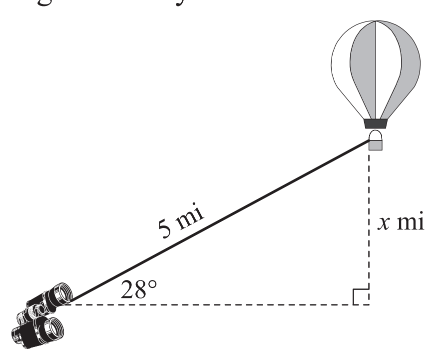
....

....

....

$x =$ *height* = ... m

f) You are observing a hot air balloon which is 5 miles away from you and makes an angle of 28° with your eye level. If sin 28°≈ 0.47, how high above eye level is the balloon?



sin 28° =

....

....

....

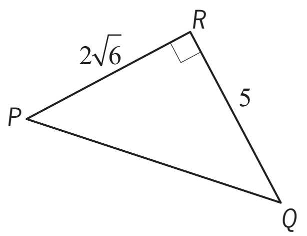
$x =$ .... mi

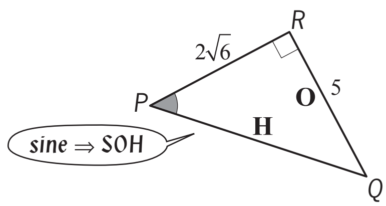
Skill 28.14 Calculating the value of trigonometric ratios in right triangles by first applying Pythagorean theorem.

- Label each side of the triangle with H, O and A.
- Apply Pythagorean theorem to calculate the unknown side length of the triangle.
  (see skill 28.3, page 333 and skill 28.4, page 334)
- Use one of the SOH - CAH - TOA relations to calculate the required trigonometric ratio.

$\sin\theta = \frac{O}{H}$ **(SOH)** $\cos\theta = \frac{A}{H}$ **(CAH)** $\tan\theta = \frac{O}{A}$ **(TOA)**

Q. Calculate the value of sin *P* in this triangle.





A. $\sin P = \frac{opposite}{hypotenuse} = \frac{RQ}{PQ}$ (unknown PQ)

$PQ^2 = PR^2 + RQ^2$ (Pythagorean theorem)

$PQ^2 = (2\sqrt{6})^2 + 5^2$

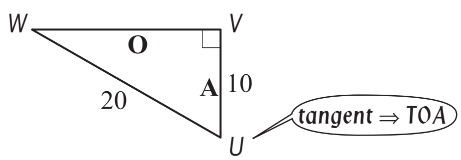
$PQ^2 = 24 + 25$

$PQ^2 = 49$

$PQ = 7$

$\sin P = \frac{RQ}{PQ} = \mathbf{\frac{5}{7}}$

a) Calculate the value of tan *U* in this triangle.
[Reduce the radical to simplest form.]



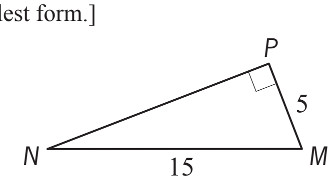
$\tan U = \frac{opposite}{adjacent} = \frac{VW}{UV}$

$VW^2 = UW^2 - UV^2$

$VW^2 = 300 \Rightarrow VW = 10\sqrt{3}$

$\tan U = \frac{VW}{UV} = \frac{10\sqrt{3}}{10}$ = ☐

b) Calculate the value of cos *N* in this triangle.
[Reduce the radical to simplest form.]



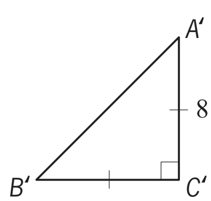
$\cos N =$ $=$

$NP^2 =$

$NP^2 =$ $\Rightarrow NP =$

$\cos N =$ $=$ ☐

c) Calculate the value of cos *B′* in this triangle.
[Reduce the radical to simplest form.]



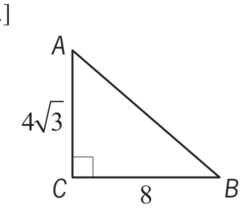
$\cos B' =$

$A'B'^2 =$

$A'B'^2 =$ $\Rightarrow A'B' =$

$\cos B' =$ $=$ ☐

d) Calculate the value of sin *B* in this triangle.
[Reduce the radical to simplest form.]



$\sin B =$ $=$

$AB^2 =$

$AB^2 =$ $\Rightarrow AB =$

$\sin B =$ $=$ ☐